Trade liberalization with mobile capital and firm heterogeneity

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Abstract

This paper develops a general equilibrium model that captures the delocation of firms, imbalance of trade in goods, and firm selection into export markets. We show that, in the presence of capital being mobile across countries and firms being heterogeneous in productivity, the spatial inequalities in wages and firm allocations are differently affected by trade liberalization. We explore the welfare effects of trade through two channels: changes in the share of expenditure on domestic goods and factor price adjustments to the spatial reallocation of capital. Selection into export markets magnifies the impact of mobile capital on the welfare gains from trade.

Keywords: mobile capital, firm heterogeneity, international inequalities, gains from trade

JEL Classification: R12, R13, F12, F15

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1 Introduction

Globalization affects countries via both goods trade and capital movement. In spite of this widespread perception, the role of capital mobility has received little attention in the recent literature. While new theories of trade successfully account for the feature of firm selection into exporting, existing studies typically assume that capital, as a production factor, is spatially immobile. Besides this assumption, recent studies on the welfare effects of trade require that trade in goods is balanced. Empirical observations, however, indicate that capital movement interacts with goods trade in shaping international inequalities, and trade in goods is generally unbalanced between countries.

This paper explores how mobile capital affects the impacts of trade liberalization on international inequalities and welfare, in a Melitz (2003) model with firm heterogeneity. We introduce two factors of production: labour and capital. The former is immobile between countries, whereas the latter is spatially mobile. Returns on capital investments are repatriated to the country in which capital owners live. Under this setup, a country’s gross domestic product (GDP) is not necessarily equal to its total income, and the imbalance of goods trade occurs with country asymmetry in the size of capital flows. We analyse how this important feature shapes international inequalities and study the corresponding welfare implications.

Our theoretical analysis considers a simplified structure with two countries engaging in trade within a monopolistically competitive sector. Countries differ in market sizes. Factor prices, such as wage and rental rates, endogenously adjust to changes in trade costs and capital movement. Since entry involves the usage of mobile capital, there exists a firm delocation effect that changes the mass of entrant firms in each country. Meanwhile, the existence of firm heterogeneity implies a firm selection effect that determines the cutoff productivity levels above which an entrant firm stays in the industry or a producing firm exports to the foreign market.

We find that the international inequalities in wages and firm allocations respond differently to trade liberalization. Specifically, when the variable trade cost decreases, the wage gap between countries is either bell-shaped or monotonically increasing, whereas the share of producing firms monotonically increases in the larger country. These results are explained by the following two facts. First, for the larger country, capital mobility magnifies its size advantage through the firm delocation effect. The wage gap increases if trade liberalization induces more capital inflows and firm entry bids up the wage rate of
local workers.

Second, firm heterogeneity plays two additional roles in the changes in international inequalities under trade liberalization. On the one hand, it magnifies the effects of market size and trade liberalization on the distribution of economic activities. This result arises from the intra-industry reallocation effect, which lowers the cutoff productivity for firm survival in the larger country relative to that in the smaller one. On the other hand, the intensity of firm selection into exporting affects the benefit a country gets when there is an improvement in access to foreign markets. In the case of less firm heterogeneity, as represented by a larger shape parameter of the productivity distribution, an entrant is more likely to draw a low productivity from the distribution. This leads to a lower share of exporting firms, together with a smaller improvement in foreign market access for the smaller country. Hence less firm heterogeneity leads to a monotonically increasing wage gap, whereas greater firm heterogeneity raises the smaller country’s benefit from trade liberalization and results in a bell-shaped wage gap.

We then explore the welfare implications of mobile capital. In a seminal paper, Arkolakis, Costinot, and Rodríguez-Clare (2012, henceforth ACR) derive a sufficient statistic of welfare that includes the share of expenditure on domestic goods and the elasticity of trade with respect to variable trade costs. This statistic requires trade in goods to be balanced between countries, which may not hold in the presence of mobile capital. We derive a measure of gains from trade when trade flows coexist with capital reallocation across countries.

We link the welfare effects of capital reallocation to endogenous changes in the relative factor prices. As more capital flows into a country, the demand for labour in production is larger, which hence implies an increase in the local wage-rent ratio. Conversely, the wage-rent ratio decreases if capital flows out of a country. Our welfare analysis reveals that, given the share of expenditure on domestic goods, the larger country gains more from trade, due to an inflow of capital and a higher wage-rent ratio. For the smaller country, however, the welfare implication of capital mobility is ambiguous. Responding to a smaller wage-rent ratio when trade is opened, the local welfare may either increase or decrease. Put differently, while the mobility of capital improves the efficiency of its allocation between countries and increases the return to capital, the resulting changes are particularly beneficial to a country if it is a net importer of capital services.

Based upon these theoretical results, we then numerically examine how capital mobil-
ity matters for the effects of trade liberalization. We start with a counterfactual exercise that restricts the mobility of capital and thus the equalization of rental rates across countries, while fixing other exogenous parameters like factor endowments, trade costs, and firm productivity distributions. We show that the removal of capital mobility leads to the absence of the firm delocation effect and the alleviation of agglomeration forces that attract firms to the larger country. Meanwhile, by changing the bilateral balances in goods trade, the removal of capital mobility generates a monotonically decreasing wage gap between countries, since the smaller country now gains more from improved access to the larger foreign market.

Therefore, our counterfactual exercise indicates a relationship between capital mobility and the magnification of international inequalities under trade liberalization. The result on changes in relative factor price reconciles with the fact that even among developed countries with similar factor endowments or productivity distributions, trade liberalization does not necessarily result in a monotonic shrinking of income gap across countries. For instance, despite gradual reductions in the bilateral trade cost, the per-capita income difference between Canada and the US has continued to increase since the 1980s. Meanwhile, in countries like Germany or Finland, it is far from obvious that trade costs with the US is negatively related to the income differences in a monotonic way.¹

Finally, our numerical exercise reveals that capital mobility is quantitatively meaningful for the welfare effects of trade. In the baseline scenario with mobile capital, change in relative factor price accounts for a sizeable share of the welfare gains from trade, and the welfare impact of change in relative factor price is amplified when firms face tougher selection into exporting. Moreover, by comparing the welfare gains from trade between the case with mobile capital and the one without mobile capital, we also find large differences in the welfare changes, and the differences are amplified when selection into exporting is tougher. Hence both capital mobility and firm heterogeneity play quantitatively relevant roles in the welfare consequences of trade liberalization.

Our paper contributes to three bodies of work. First, by capturing the mobility of production factors across countries, we complement the trade literature that explores the effects of trade liberalization on asymmetric countries by one-sector one-factor models with heterogeneous firms, including Arkolakis et al. (2008), Demidova and Rodríguez-Clare (2013), Felbermayr et al. (2013), and Behrens et al. (2014). These papers assume

¹See the online appendix for detailed empirical facts.
that production involves only one input—labour—that is immobile across countries and that trade in goods is balanced between countries. One conclusion from these papers is that the wage difference across countries decreases monotonically with trade liberalization. We differ by introducing internationally mobile capital as an additional production factor. This enables us to capture the non-monotonic changes in income difference.

Second, our analysis is related to the economic geography literature that studies spatial configuration under firm heterogeneity, including Baldwin and Okubo (2006), Okubo et al. (2010), Ottaviano (2012), von Ehrlich and Seidel (2013), and Behrens and Robert-Nicoud (2014). The main focus of this line of research is how firm heterogeneity changes the impact of trade costs on industrial agglomeration. We contribute by investigating the welfare implications of firm heterogeneity and linking the welfare consequences of factor mobility to the adjustment in relative factor prices. Our welfare analysis relates to Redding (2016), who examines the gains from trade with labour mobility within a country. Firms are homogeneous but workers are heterogeneous in this setup. We differ by considering a context of factor mobility across countries and exploring its interactions with firm heterogeneity. Our approach of modelling capital mobility is related to that of Takahashi et al. (2013), while our analysis admits their results as a special case of firm productivity distribution. Moreover, we show that the incorporation of firm selection into exporting yields different implications for the effects of trade on international inequalities and the welfare impacts of mobile capital.

Finally, we extend the traditional literature that examines whether goods trade and capital movement are substitutes or complements (e.g., Mundell, 1957; Jones, 1967;...
Markusen, 1983). This strand of research is based on perfect competition and free trade in goods. We differ by allowing for costly trade in goods and increasing returns to scale in production. By examining the implications of capital mobility for international inequalities and aggregate welfare changes, our approach complements the work by Egger and Greenaway (2011), who show that the mobility of capital matters for the value and pattern of international trade.

The remainder of this paper is organized as follows. Section 2 presents the baseline model, and Section 3 discusses the equilibrium. Section 4 examines the impacts of trade liberalization on wages and firm shares. We also present an alternative setting with different input usages. The welfare implications of capital mobility are studied in Section 5. Section 6 concludes the paper.

2 The model

2.1 Preferences

There are two countries indexed by i or j (i, j = 1, 2). Country j is populated by \( L_j \) workers, each supplying one unit of labour inelastically and receiving wage income \( w_j \). Let \( L = L_1 + L_2 \) be the total population size. Assume that country 1 is larger and that \( \theta = L_1/L > 1/2 \) denotes country 1’s share of workers. Each worker is endowed with one unit of capital. The endowment of capital in country j is \( K_j = L_j \), and the total amount of capital is \( K = L \). Workers cannot move across countries, whereas capital is perfectly mobile.\(^4\)

With two sources of income, wage and capital investment revenue, each worker consumes differentiated goods provided by domestic and foreign firms. Preferences are given by

\[
U_j = \left( \sum_{i=1,2} \int_{\omega \in \Omega_{ij}} m(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1,
\]

where \( \Omega_{ij} \) is the set of varieties produced in country i and consumed in country j, \( m(\omega) \) is the quantity of variety \( \omega \) consumed, and \( \sigma > 1 \) is the elasticity of substitution.

Let \( p_{ij}(\omega) \) be the delivered price of a variety \( \omega \) from country i to country j and \( q_{ij}(\omega) \) be the quantity demanded in country j. Utility maximization indicates that the total

\(^4\)We allow for factor endowment difference in Section 5, and explore how it affects the welfare implications of capital mobility.
demand is
\[ q_{ij}(\omega) = p_{ij}(\omega)^{-\sigma} X_j P_j^\sigma - 1, \]
where \( X_j \) is the total expenditure in country \( j \) and \( P_j \) is the price index given by \( P_j = \left[ \sum_{i=1,2} \int_{\omega \in \Omega_{ij}} p_{ij}(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)} \).

2.2 Production

There is one sector with a continuum of firms under monopolistic competition. Each firm produces a differentiated variety \( \omega \) with productivity \( \varphi \). There are two factors of production, labour and capital. In the baseline model, capital is the fixed input while labour is the variable input. Following Melitz (2003), firms are ex ante identical. After locating in country \( i \) and renting \( f_e \) units of capital as the sunk costs of entry, each entrant observes its idiosyncratic productivity \( \varphi \). Assume that \( \varphi \) is randomly drawn from a cumulative distribution \( G(\varphi) \) that is symmetric between countries.

Once an entrant in country \( i \) stays in the market and produces with productivity \( \varphi \), selling \( q_{ij} \) units of output to country \( j \) requires \( f_{ij} > 0 \) units of capital and \( \ell_{ij}(\varphi) = \tau_{ij} q_{ij}(\varphi)/\varphi \) units of labour. A larger \( \varphi \) implies a smaller marginal cost. The amount of fixed input \( f_{ij} \) is \( f_{ij} = f_d \) if \( i = j \) and \( f_{ij} = f_x > f_d \) if \( i \neq j \). Namely, domestic production involves \( f_d \) units of capital as the fixed input while exporting involves \( f_x \) units of capital. Both \( f_x \) and \( f_d \) are exogenous constants. Assume also that the variable trade cost takes the iceberg form and is symmetric between countries where \( \tau_{ij} = \tau_{ji} = \tau \geq 1 \) if \( j \neq i \) and \( \tau_{ij} = 1 \) if \( j = i \). Denote by \( w_i \) and \( r_i \) the wage rate and rental rate in country \( i \), respectively. The total cost equals
\[ TC_{ij}(\varphi) = r_i f_{ij} + w_i \tau_{ij} q_{ij}(\varphi)/\varphi. \]
We set labour in country 2 as the numeraire \((w_2 = 1)\) and denote by \( w \) the (relative) wage rate of country 1, which measures the wage gap between countries.

Given the total demand \( q_{ij}(\varphi) \) in (1), the profit-maximizing price is a constant markup over the delivered marginal cost: \( p_{ij}(\varphi) = \sigma w_i \tau_{ij} / [\sigma - 1] \varphi \). The profit of selling from country \( i \) to country \( j \) is \( \sigma^{-1} X_j P_j^{\sigma-1}(p_{ij}(\varphi))^{1-\sigma} - r_i f_{ij} \). Since the profit increases with \( \varphi \), there exists a productivity cutoff \( \varphi^*_{ij} \) above which a firm sells from country \( i \) to country \( j \):
\[ \varphi^*_{ij} = \frac{\sigma}{\sigma - 1} \frac{w_i \tau_{ij}}{P_j} \left( \frac{\sigma r_i f_{ij}}{X_j} \right)^{1/\sigma}. \]
Following the literature, we restrict exogenous parameters to ensure that $\varphi_{ij}^* > \varphi_{ii}^*$ always holds when $j \neq i$. That is, exporters are more productive than firms that serve their domestic markets only. Equation (3) also determines a domestic productivity cutoff $\varphi_{ii}^*$ below which an entrant in country $i$ exits the market.

Conditional on selling from country $i$ to country $j$, the ex post distribution of firm productivity $\mu_{ij}(\varphi)$ is

$$
\mu_{ij}(\varphi) = \begin{cases} 
\frac{g(\varphi)}{1 - G(\varphi_{ij}^*)}, & \text{if } \varphi \geq \varphi_{ij}^*, \\
0, & \text{otherwise},
\end{cases}
$$

where $g(\varphi)$ is the probability density function of $\varphi$. Following Melitz (2003), the ex post weighted average productivity is defined as

$$
\bar{\varphi}_{ij} = \left[ \int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} \mu_{ij}(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}.
$$

(4)

Following the literature, we introduce the Pareto distribution of $\varphi$ such that $G(\varphi) = 1 - \varphi^{-\kappa}$, where $\varphi \geq 1$ and $\kappa > \max\{1, \sigma - 1\}$ is the shape parameter. A smaller $\kappa$ corresponds to greater heterogeneity of firm productivity. Given this distribution, the weighted average productivity (4) is

$$
\bar{\varphi}_{ij} = \left( \frac{\kappa}{\kappa - \sigma + 1} \right)^{1/(\sigma-1)} \varphi_{ij}^*.
$$

(5)

The condition of $\kappa > \sigma - 1$ ensures that $\bar{\varphi}_{ij}$ is well defined. The price index is rewritten as

$$
P_j = \left[ \frac{\kappa}{\kappa - \sigma + 1} \sum_i M_i^e (\varphi_{ij}^*)^{\sigma-1-\kappa} (w_i \tau_{ij})^{1-\sigma} \right]^{\frac{1}{1-\sigma}},
$$

(6)

where $M_i^e$ is the mass of entrants in country $i$.

3 Market equilibrium

3.1 Equilibrium conditions

We now consider the equilibrium conditions. First, free entry and exit equate the sunk costs of entry to the ex ante expected profit. For country $i$, this free entry condition

\footnote{When country asymmetry is sufficiently large or the costs related to exporting are sufficiently low, it is possible that firms in the smaller country can easily export goods to the larger foreign market. In this case, the productivity cutoff of exporting is smaller than that of domestic production. We exclude this case by assuming that the population asymmetry satisfies the conditions analysed in Appendix E.}
becomes
\[ r_{i}f_{e} = \frac{\sigma - 1}{\kappa - \sigma + 1} \sum_{j} (\phi_{ij}^{*})^{-\kappa} r_{i}f_{ij}, \tag{7} \]

where \( r_{i}f_{e} \) is the sunk cost of entry, \((\phi_{ij}^{*})^{-\kappa} = 1 - G(\phi_{ij}^{*})\) is the probability of selling from country \( i \) to country \( j \), and \( r_{i}f_{ij}[(\phi_{ij}/\phi_{ij}^{*})^{\sigma} - 1] = r_{i}f_{ij}(\sigma - 1)/(\kappa - \sigma + 1) \) is the expected profit by (5).

Second, the goods markets clear. Denote by \( X_{j} \) the total expenditure of country \( j \) and by \( \pi_{ij} = X_{ij}/X_{j} \) the share of country \( j \)'s expenditure on goods from country \( i \). Since the returns on capital investment at the world market are repatriated to the country of capital owners, \( X_{j} = w_{j}L_{j} + \tau K_{j} \) holds, where the total expenditure \((X_{j})\) is equal to the total income of workers (who are also capital owners) and \( \tau \) is the worldwide rental rate of capital. The total firm revenue in country \( i \) \((R_{i})\) is the sum of firm revenue across all the markets: 
\[ R_{i} = \sum_{j} \pi_{ij}X_{j}. \]

Third, consider the conditions that ensure the clearing of factor markets. Within each country, workers are employed by the producing firms as variable inputs. The cost function in equation (2) and the CES demand then indicate that the total wage payment \((w_{i}L_{i})\) is a constant fraction of the total firm revenue \((R_{i})\), such that
\[ w_{i}L_{i} = \frac{\sigma - 1}{\sigma} R_{i} = \frac{(\sigma - 1)\kappa}{\kappa - \sigma + 1} \sum_{j} M_{ij}r_{i}f_{ij} \tag{8} \]
holds under the Pareto distribution of \( \varphi \), where \( M_{ij} = M_{i}^{*}(\phi_{ij}^{*})^{-\kappa} \) is the mass of firms selling from country \( i \) to country \( j \).\(^6\) Capital, on the other hand, is used as the sunk cost of entry and the fixed cost of domestic production and exporting. Hence the total capital rent payment is \( M_{i}^{c}r_{i}f_{e} + \sum_{j} M_{ij}r_{i}f_{ij} \). Multiplying both sides of equation (7) by \( M_{i}^{c}r_{i} \), we obtain
\[ M_{i}^{c}r_{i}f_{e} + \sum_{j} M_{ij}r_{i}f_{ij} = \frac{\kappa}{\kappa - \sigma + 1} \sum_{j} M_{ij}r_{i}f_{ij}, \tag{9} \]
where the LHS represents the rent payment by firms in country \( i \). Equations (8) and (9) then imply that the ratio of wage payment to capital rent payment is fixed at \( \sigma - 1 \), which holds for all the countries.

\(^6\)The expected revenue of a firm that sells from \( i \) to \( j \) is \( r_{i}f_{ij}\sigma\kappa/(\kappa - \sigma + 1) \) according to (5). Multiplying this term by the mass of firms \( M_{ij} \) and summing across destination markets, the total revenue \( R_{i} \) equals \( \sum_{j} M_{ij}r_{i}f_{ij}\sigma\kappa/(\kappa - \sigma + 1) \), which implies the second equality in (8).
Aggregating across countries, the world capital market clearing condition indicates that

\[ K = \sum_i M_i^e \bar{f}_e + \sum_i \sum_j M_{ij} f_{ij} \]  

(10)

where the worldwide endowment of capital \( K \) is used by firms for entry and production. With perfect capital mobility, the rental rates are equalized across countries such that \( r_i = \bar{r} \) holds and the equilibrium capital rent rate \( \bar{r} \) is determined by

\[ \bar{r} = \frac{\sum_i w_i L_i}{(\sigma - 1)K} = \frac{w\theta + 1 - \theta}{\sigma - 1}. \]  

(11)

The second equality in (11) reveals that \( \bar{r} \) depends on country 1’s relative wage rate \( w \) and its population share \( \theta \) under our two-country setup. From equations (9) and (10),

\[ M^e \equiv \sum_i M_i^e = \frac{(\sigma - 1)K}{\kappa f_e} \]  

(12)

holds so that the worldwide mass of entrants \( M^e \) is constant in equilibrium.

Finally, we examine how capital mobility affects goods trade in each country. With repatriation of capital investment to the country where immobile workers live, the following condition holds in equilibrium:

\[ (M_i^e \bar{r} f_e + \sum_j M_{ij} \bar{r} f_{ij}) - \bar{r} K_i = R_i - X_i. \]  

(13)

Equation (13) says that any net inflow of capital payment in country \( i \) (the LHS) should be offset by its surplus in goods trade (the RHS), which is also the difference between total firm revenue and total consumer spending. Here, the rental rate \( \bar{r} \) is determined by (11), and hence the volumes of capital flows and goods trade are endogenously determined.

### 3.2 Implications of market size and firm heterogeneity

We now discuss how market size asymmetry and firm selection into exporting may affect the cross-country differences in wage rates, productivities and shares of firms. First, equations (8) and (9) imply that the wage payment in country \( i \) equals \( w_i L_i = M_i^e \bar{r} \kappa f_e \) under mobile capital.\(^7\) Comparing between countries, we obtain \( M_i^e/M_j^e = w_i L_i/(w_j L_j) \).

\(^7\)Our baseline model involves labour as the variable input in production. Given CES demand, the wage payment is a constant fraction of firm revenue \( (w_i L_i = \frac{\sigma - 1}{\sigma} R_i) \). With Pareto distribution of productivity, the aggregate revenue is proportionate to the sunk costs of entry \( (R_i = \frac{\sigma m}{\sigma - 1} M_i^e r_i f_e) \). Meanwhile capital mobility equalizes the rental rate across countries \( (r_i = \bar{r}) \).
That is, the ratio of the mass of entrants is equal to that of wage payment between countries. The mass of entrants in country $i$ is then expressed as

$$M^e_i = \frac{w_i L_i}{\sum_j w_j L_j} M^e,$$

where $M^e$ is given by (12). Equation (14) captures the firm delocation effect. Namely, with capital mobility, the mass of entrants in a country $M^e_i$ increases with the local wage rate $w_i$. If $w_i$ varies with the variable trade cost $\tau$, then $M^e_i$ adjusts endogenously when trade is liberalized.

Under the two-country setup, equation (14) links $\lambda^e = M^e_1/(M^e_1 + M^e_2)$, which is country 1’s share of entrants, to its relative wage rate $w$ and its population share $\theta$:

$$\lambda^e - \theta = \frac{\theta(1 - \theta)(w - 1)}{w\theta + (1 - \theta)}.$$

Thus, if country 1 receives a more-than-proportionate share of entrants ($\lambda^e > \theta$), then its wage is relatively higher ($w > 1$). Given the market size difference, firm entry in the larger country bids up the demand for immobile labour input and thus the relative wage of workers.

With firm heterogeneity, a second implication is that market size affects the productivity cutoffs. Equation (3) links $\varphi^*_ij$ ($j \neq i$) to $\varphi^*_ii$ such that

$$\varphi^*_{12} = \varphi^*_{22}\Lambda w \quad \text{and} \quad \varphi^*_{21} = \varphi^*_{11}\left(\frac{\Lambda}{w}\right),$$

where $\Lambda \equiv \tau(f_x/f_d)^{\sigma-1}$. Meanwhile, by including (16) in the free entry condition (7), the ratio of domestic productivity cutoffs is linked to the relative wage rate $w$:

$$\left(\frac{\varphi^*_{11}}{\varphi^*_{22}}\right) = \frac{1 - w^\kappa \Delta}{1 - w^{-\kappa} \Delta},$$

where $\Delta \equiv \tau^{-\kappa}\Gamma$ is a measure of market integration that combines the variable trade cost $\tau$ and the fixed cost parameters expressed by a constant $\Gamma \equiv (f_x/f_d)^{\frac{\sigma-1}{\sigma-1}}$. For any given $\kappa$, $\sigma$, and $f_x/f_d$, trade liberalization reduces the variable trade cost $\tau$ and promotes market integration.\(^8\) As in Melitz (2003), we assume that the combination of fixed and variable trade costs satisfies $\Delta \in [0,1)$ and more productive firms become exporters. Meanwhile

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\(^8\)In reality, the fixed cost of exporting is much less likely to be reduced than the variable trade cost, due to foreign regulations like restrictions of market entry, standards of product quality, or technical barriers to trade. Hence we simply assume that trade liberalization is not realized via the reduction in $f_x$. 


(17) indicates that $\phi^*_{11} < \phi^*_{22}$ if $w > 1$, which reflects the fact that a larger market demand increases firm profitability and thus entrants with lower productivity draws are able to stay in the market.

Next, consider the shares of producing firms. Since entrants with productivity below $\phi^*_{ii}$ exit the market, the mass of producing firms in country $i$ equals $M_i = M_i^e [1 - G(\phi^*_{ii})]$. Let

$$\lambda = \frac{M_1}{M_1 + M_2} = \frac{1}{1 + \left(\frac{1}{\lambda^e} - 1\right)\left(\frac{\phi^*_{11}}{\phi^*_{22}}\right)^\kappa}$$

(18)

be country 1’s share of producing firms. The second equality of (18) shows that $\lambda$ is affected by both the firm delocation effect (which changes $\lambda^e$) and the firm selection effect (which changes $\phi^*_{11}/\phi^*_{22}$). By combining (15) and (17), we rewrite $\lambda$ as a function of $w$:

$$\lambda = \frac{w \theta (1 - w^{-\kappa} \Delta)}{w \theta (1 - w^{-\kappa} \Delta) + (1 - \theta)(1 - w^\kappa \Delta)}. \quad (19)$$

The first part of Appendix D shows a positive relationship between $\lambda$ and $w$. Intuitively, the firm delocation effect implies a more-than-proportionate share of entrants in the larger country, and the firm selection effect amplifies the resulting increase in labour demand, which further bids up the wage rate of local workers.

Finally, we pin down country 1’s equilibrium relative wage rate $w^*$ via the following equation, which is derived in Appendix A:

$$F(w, \Delta) \equiv A_2(w) \Delta^2 + A_1(w) \Delta + A_0(w) = 0, \quad (20)$$

where $A_2(w) = \theta (\sigma - 1 + \theta) w - (1 - \theta)(\sigma - \theta)$, $A_1(w) = \sigma[(1 - \theta) w^\kappa - \theta w^{1-\kappa}]$, and $A_0(w) = \theta (1 - \theta)(w - 1)$.

Several properties are known from (20), and we summarize them in Lemma 1.

**Lemma 1.** $F(w, \Delta)$ increases with $w$. For any $\Delta \in [0, 1)$, Equation (20) yields a unique equilibrium wage rate $w^*$ satisfying $w^* \in [1, \min\{\Delta^{-\frac{1}{\kappa}}, (\frac{\theta}{1-\theta})^{-\frac{1}{\kappa-1}}\}]$. Besides, $w^*$ increases with $\theta$.

**Proof.** See Appendix B. \hfill \Box
Note that if $\kappa \to \sigma - 1$, then Equation (20) degenerates to the wage equation in Takahashi et al. (2013) where firms are homogeneous in productivity. This result is consistent with Felbermayr et al. (2013, Section 4.2.1) who find that the channel of firm selection is shut down when $\kappa \to \sigma - 1$, since it generates a significantly large dispersion of firm productivity, and a tiny mass of very productive firms accounts for the majority of production in the industry. Meanwhile these infra-marginal firms always export, which implies the absence of the within-industry reallocation effect.

Furthermore, Lemma 1 implies the following inequalities when $\Delta \in (0, 1)$: (a) $w^* > 1$, (b) $\lambda^* > \lambda^e > \theta > 1/2$, and (c) $\varphi_{11}^* < \varphi_{22}^*$. When trade is costly, the larger country has a higher relative wage rate, its shares of entrants and producing firms are more than proportionate, and firm heterogeneity strengthens the concentration of firms in the larger market.\footnote{These results can be linked to the literature on home market effects: (i) the positive correlation between wage rate and market size is theoretically explored in Krugman (1980) and widely verified in empirical work including Redding and Venables (2004), Hanson (2005), among others; and (ii) the result of more-than-proportionate firm shares in large countries is in line with Helpman and Krugman (1985) and Martin and Rogers (1995). Takahashi et al. (2013) provide a synthesis by allowing for capital mobility and wage adjustments to trade. Unlike their work, our analysis on firm heterogeneity indicates that it amplifies the role of market size in international inequalities. Moreover, our welfare analysis in Section 5 shows that the relationship between capital flow and market size has important implications for the welfare effects of trade.}

By allowing for mobile capital, an additional implication is that the larger country is also the net exporter of goods and a net importer of capital, with the directions and volumes of capital flows and goods trade being related to $w^*$ and $\bar{r}^*$ in equilibrium.\footnote{Appendix C provides the detailed proofs. Note that market size asymmetry is the only driving force for the imbalances in goods trade and capital flows here. By allowing for additional asymmetries such as factor endowment differences, it is likely that a larger country may run a deficit in goods trade. See Section 5 for the related discussions.}

4 Impacts of trade liberalization

This section studies how the wage and firm share differences respond to trade liberalization. We start by using (20) to explore the change in country 1’s equilibrium relative wage rate $w^*$. We then graphically examine how country 1’s equilibrium share of producing firms $\lambda^*$ may change.
4.1 Change in the relative wage rate

Note that Equation (20) is well defined for $\Delta \in [0, 1]$. Taking the partial derivatives of (20) with respect to $\Delta$ yields

$$
\frac{dw^*}{d\Delta} \bigg|_{\Delta=0} = \frac{\sigma(2\theta - 1)}{\theta(1 - \theta)} > 0, \quad \frac{dw^*}{d\Delta} \bigg|_{\Delta=1} = \frac{1 - 2\theta}{\kappa} < 0.
$$

(21)

Because (20) is a quadratic function of $\Delta$, it has at most two roots of $\Delta$ for a given $w^*$. Put differently, any horizontal line crosses the wage curve $w^*(\Delta)$ at most twice in the $\Delta$-$w^*$ plane. Given (21), it indicates that the curve of $w^*(\Delta)$ is bell shaped. Hence there exists a threshold $\tilde{\Delta} \in (0, 1)$ at which $\partial w^* / \partial \Delta$ changes its sign from positive to negative.

Since $\Delta$ is negatively related to the variable trade cost $\tau$ for any given $\sigma$, $\kappa$ and $\Gamma$, we are left with two possible patterns of the equilibrium wage gap $w^*$ under trade liberalization. For large values of $f_x/f_d$ and $\kappa$, a reduction in $\tau$ magnifies the wage gap. In contrast, for small values of $f_x/f_d$ and $\kappa$, there is a bell-shaped relationship between $\tau$ and the wage gap.

To provide an intuitive explanation, we consider how $w^*$ adjusts if the variable trade cost $\tau$ increases from $\tau = 1$. Suppose that $f_x > f_d$ and selection into export markets works. An increase in $\tau$ generates two opposing forces. The first arises from the market access effect. Exporting becomes more difficult, which raises the productivity cutoff of exporting and decreases the mass of exporters. Since countries are asymmetric in $w_i$ and $P_i$ ($w_1 > w_2$ and $P_1 < P_2$) when $f_x > f_d$, firms in the smaller country are more negatively affected by the reduction in market access. Thus a larger drop in labour demand occurs in country 2, which increases country 1’s wage rate. This impact on wages is stronger when the proportion of exporters is larger, which occurs if the fixed exporting cost is smaller (lower $f_x/f_d$), or when the likelihood of a productivity draw necessary for exporting is higher (lower $\kappa$).

Besides the reduction in market access, weakened market competition works as the other force that affects $w^*$. Imported varieties become more expensive as $\tau$ increases. The resulting weaker import competition raises domestic profit and increases labour demand. Given the asymmetry of productivity cutoffs, the smaller country is more protected from

\[\text{If } \Delta = 1, \text{ then } w^* = 1 \text{ holds. In this case, there is free trade } (\tau \to 1) \text{ and the channel of selection into exporting is shut off } (\kappa \to \sigma - 1), \text{ hence factor price equalization realizes.}\]

\[\text{A decline in } \tau \text{ from autarky } (\tau \to \infty) \text{ necessarily increases country 1’s relative wage. As trade occurs, the larger country receives a net inflow of capital, and its wage rate is strictly higher than that under autarky.}\]
weakened competition. This implies a larger labour demand in country 2 and decreases
country 1’s relative wage rate. The impact on wages is stronger if the fixed exporting cost
is larger (higher $f_x/f_d$) or the dispersion of firm productivity is smaller (higher $\kappa$).\(^{14}\)

To summarize, when $f_x/f_d$ or $\kappa$ is large, the proportion of exporting firms is small.
This leads to a strong market competition effect and a weak market access effect. Trade
liberalization then results in a monotonically increasing wage differential. However, if
$f_x/f_d$ and $\kappa$ are small, the proportion of exporting firms is large. As trade liberalizes, the
market access effect gradually gets stronger, which finally leads to wage convergence.

Our setup reveals that the wage gap does not shrink monotonically if the variable
trade costs decrease. This result differs from those of the existing one-factor, one-sector
trade models without factor movement (e.g., Krugman, 1980; Arkolakis et al., 2008;
Behrens et al., 2014). In these models, the wage gap decreases monotonically with trade
liberalization. This result is not affected by either firm heterogeneity or the variable price
elasticity of demand.\(^{15}\) One feature in these models is that trade in goods is balanced and
the mass of entrants in each country is fixed. The smaller country then benefits more
from exporting to the larger country when trade is liberalized.

Our results of wage gap changes differ for the following reasons. First, the firm delo-
cation effect links the mass of entrants in one country to its wage rate, both of which vary
with the variable trade cost $\tau$. Capital mobility magnifies the larger country’s size ad-
vantage by creating a more-than-proportionate share of entrants. Second, with selection
into export markets, $f_x/f_d$ and $\kappa$ affect the impact of $\tau$ on wages. If the proportion of
exporting firms is small (corresponding to high values of $f_x/f_d$ or $\kappa$), even a large decrease
in $\tau$ cannot result in a sufficient increase of labour demand in the smaller country, so that
the wage gap keeps expanding.

4.2 Change in firm shares

We now analyse the effect of trade liberalization on country 1’s equilibrium share of
producing firms $\lambda^*$. Note that equations (13) and (19) imply an equation system that
determines country 1’s relative wage $w$ and its firm share $\lambda$ in equilibrium. We plot the

\(^{14}\)In contrast, when $f_x \to f_d$ or $\kappa \to \sigma - 1$, countries are affected symmetrically by weaker market
competition when $\tau$ increases from $\tau = 1$.

\(^{15}\)Arkolakis et al. (2008) consider firm heterogeneity, while Behrens et al. (2014) allow for both variable
price elasticity of demand and income effect.
relationships between $w$ and $\lambda$ of each equation in Figure 1. Since Section 4.1 reveals that a smaller $\tau$ may either increase or decrease country 1’s equilibrium relative wage $w^*$, we focus on the change in country 1’s equilibrium share of producing firms $\lambda^*$ on the vertical axis. From (19), which is related to the capital mobility condition, we obtain a positively sloped curve between $w$ and $\lambda$ in Figure 1. The positive slope arises because a higher $w$ induces a more-than-proportionate share of entrants in country 1 ($\lambda^e > \theta$), and firm heterogeneity further increases the share of producing firms in country 1 ($\varphi^*_{11} < \varphi^*_{22}$).

![Diagram showing relationships between $w$, $\lambda$, and $\lambda^*$](image)

**Figure 1:** Trade liberalization increases the larger country’s firm share $\lambda^*$

Meanwhile, the balance of payment condition in Equation (13) can be rewritten as a function of $w$ and $\lambda$ (see the online appendix for details):

$$\Delta \left[ \theta w^{1-\kappa} - (1 - \theta) \lambda^{\kappa-1} \frac{1 - \theta}{\theta} \frac{\lambda}{1 - \lambda} \right] = \theta(w - 1) \frac{1 - \theta + \theta w^{1-\kappa} \Delta}{\theta w + \sigma - \theta}. \quad (22)$$

The second half of Appendix D further shows that $\lambda$ decreases with $w$ in (22). From (18), $\lambda$ is positively related to $\lambda^e$ but negatively related to $\varphi^*_{11}/\varphi^*_{22}$. As $w$ increases, the larger country exports more. The resulting intra-industry reallocation effect (which increases $\varphi^*_{11}/\varphi^*_{22}$) dominates the firm delocation effect (which increases $\lambda^e$), so that a negative relationship between $\lambda$ and $w$ arises.

---

16The net export of country 1 is $\bar{r}K(\lambda^e - \theta)$, which is equal to the size of net inflow of capital to country 1. Both $\bar{r}$ and $\lambda^e$ increase with $w$, hence country 1 should export more as it receives a larger size of capital inflow.
Appendix D also demonstrates that both the capital mobility and the balance of payment curves shift up in Figure 1 (moving from the solid to the dashed curves) when trade is liberalized. Therefore, country 1’s equilibrium share of producing firms $\lambda^*$ increases. According to (18), at any given $w$ and $\lambda^c$, $\lambda$ is negatively related to $\varphi_{11}/\varphi_{22}$. Thus, the upward shifts of both curves indicate a decrease in $\varphi_{11}/\varphi_{22}$. In this case, trade liberalization makes it easier for exporters in the smaller country to access the larger foreign market, which results in a smaller value of $\varphi_{11}/\varphi_{22}$ via the intra-industry reallocation effect.

Therefore, firm heterogeneity indicates a difference in the effects of trade liberalization on the spatial inequalities in wages and firm allocations. On the one hand, the non-monotonic relationship between $\Delta$ and $w^*$ implies that trade liberalization is likely to be associated with a smaller $w^*$ and $\lambda^*$. In this case, entrants delocate out of the larger country, and the wage gap between countries shrinks. On the other hand, with the monotonic increase in $\lambda^*$, the impact of a reduction in $\varphi_{11}^*/\varphi_{22}^*$ dominates that of entrant delocation, so that there is a concentration of producing firms in the larger country.\footnote{When the initial values of $\tau$, $f_x/f_d$ and $\kappa$ are high, trade liberalization may result in larger $w^*$, $\lambda^c*$, and $\lambda^*$. In this case, firm heterogeneity strengthens the entrant delocation effect, leading to a larger difference of producing firm shares.}

Here we distinguish entrants from producing firms and capture the differences in firm share changes at each margin. This differs from Takahashi et al. (2013)’s work, where the absence of firm heterogeneity results in a non-monotonic change in the larger country’s firm share.

We summarize the results as follows:

**Proposition 1.** When the variable trade cost $\tau$ decreases: (i) the relative wage of the larger country exhibits two possible patterns of change: either it increases when $f_x/f_d$ or $\kappa$ is large, or it is bell shaped when $f_x/f_d$ or $\kappa$ is small; (ii) the larger country’s share of producing firms increases.

### 4.3 Discussions

#### 4.3.1 The implications of capital mobility

We have shown that the presences of mobile capital and firm heterogeneity result in different changes in cross-country asymmetries in wage rates and firm shares under trade
liberalization. Now we explore how the results for international inequalities are affected by the mobility of capital, by examining the change in model equilibrium when capital becomes internationally immobile. Two striking features stand out if we let capital to be immobile. First, at the country level, trade in goods is now balanced in equilibrium, and a country’s total value of output $R_i$ equals its total value of expenditure $X_i$.\(^{18}\) Second, given the CES demand and the specification of cost function in our baseline model setup, the equilibrium relative factor price \((w_i/r_i)\) is constant at \((\sigma - 1)/\sigma\), which is not affected by trade. This result changes when capital is mobile, since equation (10) indicates that the equilibrium relative wage rate \(w\) and rental rate \(\bar{r}\) are both affected by the levels of trade costs.\(^{19}\) We will show in Section 5 that changes in relative factor prices generate important implications for the welfare effects of trade liberalization.

Figure 2 indicates how capital mobility changes the equilibrium relative wage rates, cutoff productivities and firm shares under different values of trade costs. We fix the values of exogenous parameters \(\{\theta, \kappa, \sigma, f_x/f_d, f_e\}\), and numerically solve for \(\{w_i^*, \varphi_{ii}, \varphi_{ij}^*, \lambda, \lambda_e\}\) in equilibrium under scenarios of immobile and mobile capital, respectively.\(^{20}\) Panel (a) plots the change in the larger country’s relative wage. In the scenario of immobile capital, the wage gap shrinks as the variable trade cost \(\tau\) falls, which reflects the fact that the smaller country benefits more from easier access to the larger foreign market. By contrast, in the scenario of mobile capital, the wage difference expands when the initial value of trade cost is large, which is in line with the results in Section 4.1. The difference in wage adjustments implies that capital movement amplifies the size advantage of a large country when trade is opened up.\(^{21}\) Similarly, panel (b) indicates that the presence of mobile capital also results in an adjustment at the entrant margin, whereas the mass of entrants in each country keeps constant if we exclude capital mobility. Since firm entry changes the mass of varieties and hence the sectoral price index in each country, which

\(^{18}\)In contrast, when capital is mobile, equation (13) allows for a difference between \(R_i\) and \(X_i\), such that the imbalance in goods trade is counteracted by flows of capital payment in the opposite direction.

\(^{19}\)The CES demand implies that the total variable inputs \((w_iL_i)\) is a constant fraction \(\frac{\sigma - 1}{\sigma}\) of total value of output \((R_i)\), and the capital payment \((r_iK_i)\) as fixed inputs is a fraction \(\frac{1}{\sigma}\) of total value of output \((R_i)\).

\(^{20}\)The exogenous parameters are set as \(\theta = 0.6, \kappa = 5, \sigma = 4, f_x/f_d = 5,\) and \(f_e = 1\) in the simulation exercise for Figure 2.

\(^{21}\)The parameter of firm heterogeneity plays an additional role since larger values of the shape parameter \(\kappa\) magnify the size advantage of the larger country, which generates an expanding wage gap when trade cost \(\tau\) keeps decreasing.
then affects firm profitability and labour demand, the adjustment of firm entry is one factor that determines the change in relative wage difference.

Panels (c) and (d) further examine the changes in cutoff productivities and firm shares. For any given variable trade cost $\tau$, we find that capital mobility raises the larger country’s relative cutoff productivity (represented by larger values of $\phi_{11}^*/\phi_{22}^*$), which implies a magnification of the intra-industry reallocation effect in the larger country. Intuitively, the presence of mobile capital incurs a more-than-proportionate share of entrant firms in the larger country, which raises demand for immobile labour and intensifies competition in the product market. The resulting reduction in expected profit of production then expels entrants with small values of productivity draws. The larger country’s share of producing firms, by contrast, is increased when we allow for the mobility of capital. This result is driven by firm entry with mobile capital, and the increase in the larger country’s firm share implies that the effect of firm entry dominates that of firm heterogeneity in the allocation of producing firms across countries.

Figure 2: Capital mobility and the effects of trade liberalization
4.3.2 The usage of capital inputs

One concern of our theoretical analysis is that in the baseline model, labour is the only variable input in production while capital is employed to cover the sunk cost of entry and the fixed costs of production. With increasing returns to scale, this setup may overstate the importance of mobile capital for international inequalities. Now we extend the model by allowing for alternative usages of capital and examining the robustness of our findings. First, following Bernard et al. (2007), we assume that entry and production involve a composite factor of production that combines capital and labour in a Cobb-Douglas form. Specifically, let the sunk costs of entry be $w_i^\alpha r_i^{1-\alpha} f_e$ ($0 < \alpha < 1$) and the cost function be $TC_{ij}(\varphi) = w_i^\alpha r_i^{1-\alpha}(f_{ij} + \tau_{ij}q_{ij}(\varphi)/\varphi)$. Second, we assume that firm entry involves labour as the only input, while production employs a composite of labour and capital in a Cobb-Douglas form. Hence the sunk cost of entry is $w_i f_e$, and the cost function is $TC_{ij}(\varphi) = w_i^\alpha r_i^{1-\alpha}(f_{ij} + \tau_{ij}q_{ij}(\varphi)/\varphi)$. In the case of perfect capital mobility, $r_i = \bar{r}$ ($i = 1, 2$) still holds in the equilibrium.\(^{22}\)

Figure 3 numerically simulates the impacts of trade liberalization on international inequalities in wages and firm allocations for the extended setup where the composite factor is used in both entry and production. Similarly, Figure A-1 of the online appendix plots the results for the other extension where entry involves immobile labour while production employs the composite factor. We put the variable trade cost $\tau$ on the horizontal axis, and put the larger country’s equilibrium relative wage ($w^*$) and its equilibrium firm share ($\lambda^*$) on the vertical axis. Two values of the shape parameter are assigned: $\kappa = 5$ and $\kappa = 8$.\(^{23}\) In both extended setups, the wage gap is bell shaped when $\kappa=5$ but monotonically increasing when $\kappa = 8$. By contrast, the larger country’s firm share is always monotonically increasing. Hence the spatial inequalities in wages and firm allocations are differently affected by trade liberalization, and Proposition 1 is qualitatively invariant to the specification of cost functions.

\(^{22}\)Appendix F lists the equations that solve the equilibrium variables in the first extended setup where the Cobb-Douglas composite factor is employed in both entry and production. For the extended setup where production involves the Cobb-Douglas composite factor while entry employs immobile labour only, see the online appendix for further discussions about the equilibrium results.\(^{23}\) The fixed cost parameters are set as $f_x/f_d = 5$, and the other parameters are set as $\theta = 0.6$, $\alpha = 0.67$, $\sigma = 4$, and $f_e = 1$.
5 Welfare analysis

This section explores how mobile capital changes the welfare impacts of trade and whether firm heterogeneity plays a role in the welfare changes. First, we link the welfare impact of capital reallocation to the change in relative factor price between mobile capital and immobile labour. Second, we examine how market size asymmetry and firm selection into exporting matter for the welfare adjustment to relative factor price changes. Third, we quantify the welfare implication of capital mobility, focusing on the contribution of relative factor price to the gains from trade and on the comparison of welfare changes under different scenarios of capital mobility. Finally, we show that the welfare impacts of capital mobility and firm heterogeneity are strengthened in the more general case with factor endowment differences.

5.1 Welfare measure

Following ACR, we first derive the expression of trade shares. Using the results of the mass of firms and productivity cutoffs, \( \pi_{ij} \) is written as

\[
\pi_{ij} = \frac{w_i L_i (w_i \tau_{ij})^{-\kappa} (f_{ij})^{\frac{\sigma-1}{\sigma-1-\kappa}}}{\sum_{s=1}^{2} w_s L_s (w_s \tau_{sj})^{-\kappa} (f_{sj})^{\frac{\sigma-1}{\sigma-1-\kappa}}}
\]  

(23)

The presence of mobile capital affects the \( \pi_{ij} \) term via changes in \( w_i L_i \), which is proportionate to the mass of entrants by equation (14). This firm delocation effect differs
from the trade models that do not include capital mobility. Meanwhile, given the Pareto distribution of \( \phi \), for \( i \neq j \), the (partial) elasticity of trade with respect to variable trade costs \( \tau_{ij} = \tau \) is \( \partial \ln(X_{ij}/X_{jj})/\partial \ln \tau_{ij} = -\kappa \).

Appendix G shows that the price index in country \( j \) can be expressed as

\[
P_j = \frac{1}{\kappa} w_j \left( \frac{R_j}{\bar{f}_e} \right) \left( \frac{X_j}{\bar{P}_d} \right)^{\frac{\kappa - \frac{\sigma + 1}{\kappa(1 - \sigma)}}{\kappa}} \psi_0,
\]

where \( \psi_0 = \sigma^{\frac{\sigma}{\sigma - 1}} (\sigma - 1)^{-1}\frac{1}{\kappa} (\kappa - \sigma + 1)^{1\frac{1}{\kappa}} \) is a constant and the share of expenditure on domestic goods \( \pi_{jj} \) is given by (23).

Equation (24) separates country \( j \)'s price index into four main terms on the RHS. The first term \( \pi_{jj}^{1/\kappa} \) shows that a smaller share of expenditure on domestic goods leads to a drop in \( P_j \). The second term \( w_j \) reflects the cost of variable inputs used in firm production. The third and fourth terms represent the effects from the entry of heterogeneous firms. Given the total firm revenue \( R_j \), each entrant rents \( f_e \) units of mobile capital as the sunk cost of entry, hence the third term indicates that \( P_j \) decreases with the mass of entrants, which is proportionate to \( R_j/(\bar{r}f_e) \). The fourth term reflects the selection effect: a higher local expenditure \( (X_j) \) decreases the cutoff productivity for domestic production and increases the mass of producing firms. As a result, \( P_j \) decreases with \( X_j \).

Since \( R_j = w_j L_j \left( \frac{\sigma}{\sigma - 1} \right) \) and \( X_j = (w_j + \bar{r})L_j \), we can further rewrite country \( j \)'s welfare \( (V_j) \) as

\[
V_j = \frac{X_j}{L_j P_j} \propto \pi_{jj}^{-\frac{1}{\kappa}} \gamma_j \chi_j,
\]

where \( \gamma_j = \frac{\bar{w}_j}{\bar{w}_j + 1} \), \( \chi_j = \frac{(\bar{w}_j + 1)^{\sigma/(\sigma - 1)}}{\bar{w}_j} \), and \( \bar{w}_j = \frac{w_j}{\bar{r}} \).

The term \( \pi_{jj}^{-1/\kappa} \) links a country’s welfare with the share of expenditure on domestic good \( (\pi_{jj}) \) and the trade elasticity \( (\kappa) \), which is identical to the ACR’s welfare formula. With mobile capital, however, equation (25) indicates that the relative factor price \( \bar{w}_j \) additionally affects the welfare measure via the \( \gamma_j^{1/\kappa} \chi_j \) term. Moreover, the change in \( \bar{w}_j \) is related to the direction of capital flows: given that labour is immobile, an inflow (outflow) of capital makes labour more (less) scare in a country, which pushes up (down) the wage-rent ratio.\(^{24}\)

---

\(^{24}\)Welfare in the composite factor setup is similar and satisfies \( V_j \propto \pi_{jj}^{-\frac{1}{\kappa}} \bar{w}_j^{\frac{\sigma - \frac{\sigma + 1}{\kappa}}{1 - \sigma}} (\bar{w}_j + 1)^{\frac{\pi_{jj} - \frac{\kappa}{\kappa - 1}}{\pi_{jj} - \frac{\kappa}{\kappa + 1}}} \), where \( \pi_{jj} \) is derived from the corresponding cost function, and the relative factor price is \( \bar{w}_j = \left( \frac{\sigma}{\bar{r}} \right) \sum_{j=1}^{N} w_j L_j / L \). See Appendix H for derivation.
Associated with any change in variable trade costs, the change in per-capita welfare is measured by

\[ \hat{V}_j = \hat{\pi}_{jj}^{-\frac{1}{\gamma_j}} \hat{\chi}_j, \]  

where as in ACR, \( \hat{x} \equiv x'/x \) denotes the change in any variable \( x \) (from \( x \) to \( x' \)). In addition to the welfare impact of change in the share of expenditure on domestic goods \( \hat{\pi}_{jj} \), capital reallocation affects welfare via the change in relative factor prices, as shown by the \( \hat{\gamma}_j \) and \( \hat{\chi}_j \) terms.\(^{25}\) Here we can link the welfare implication of factor reallocation with Redding (2016), who shows that labour reallocation is quantitatively important for a country’s welfare change under trade liberalization. We differ by analysing the flows of production factors across countries, which allows us to capture the endogenous adjustment of imbalances in goods trade and their interactions with capital flows.

5.2 Country size and the welfare implications of mobile capital

We have shown that in the presence of mobile capital, a country’s welfare is affected by the direction of capital flows. In this subsection, we use (25) and examine how the welfare impact of mobile capital is linked with a country’s market size. Consider the welfare adjustment from autarky to trade. We use superscript \( A \) to indicate the autarky equilibrium and superscript \( T \) to indicate the trade equilibrium. Compared to autarky where \( \pi^A_{jj} = 1 \), trade decreases the share of expenditure on domestic goods \( \pi^T_{jj} < 1 \), which holds for both countries. Meanwhile trade results in a higher rental rate since \( \bar{r}^T = (\theta w^T_1 + 1 - \theta)/(\sigma - 1) > \bar{r}^A = 1/(\sigma - 1) \) under our baseline setup, where countries differ in population size and country 2’s wage rate is fixed at 1. Now we check the welfare adjustments to changes in the \( \gamma_j \) and \( \chi_j \) terms, both of which are affected by the changes in relative factor price \( (\hat{w}_j) \). Under autarky, \( \bar{w}^A_1 = \bar{w}^A_2 = \sigma - 1 \) holds, whereas \( \bar{w}^T_1 > \bar{w}^A_1 \) and \( \bar{w}^T_2 < \bar{w}^A_2 \) hold when trade is at work. Together with the imbalance of goods trade, a net inflow of capital implies a larger labour demand. Thus, the wage rate in the larger country increases relative to the return to mobile capital.

\(^{25}\)The ACR welfare measure requires three macro restrictions: (a) balanced trade, (b) a constant ratio between aggregate profits and aggregate revenues, and (c) a CES import demand system. Introducing capital mobility relaxes restriction (a). Restrictions (b) and (c) are met in our setup. In addition, there exists endogenous adjustments of trade imbalances. This differs from Dekle et al. (2007), where trade imbalances are treated as exogenous.
Note that the term $\chi_j$ of (25) is minimized when $\tilde{w}_j$ equals $\sigma - 1$. Hence any deviation of $\tilde{w}_j$ from $\tilde{w}_j^A = \sigma - 1$ indicates a higher $\chi_j$, which implies that the inequality $\chi_j^T > \chi_j^A$ holds for both countries. Responding to a larger $\chi_j$, welfare in both countries increases after controlling for changes in $\pi_{jj}$ and $\gamma_j$ in (25). Intuitively, trade expands the market size and increases the rate of return on mobile capital, which yields one margin of welfare improvement.

In contrast, the welfare adjustments to change in $\gamma_j$ differ between countries. For country 1 (the larger country), $\gamma_1^T > \gamma_1^A = (\sigma - 1)/\sigma$ holds. Hence it benefits from increases in both $\chi_1$ and $\gamma_1$ when trade is opened. Given the change in the share of expenditure on domestic goods, capital mobility leads to additional welfare gains from trade for the larger country. For country 2 (the smaller country), however, the $\gamma_2$ term is smaller with trade ($\gamma_2^T < \gamma_2^A = (\sigma - 1)/\sigma$), leading to a loss of welfare. With a larger $\chi_2$ but a smaller $\gamma_2$ with trade, the net effect on country 2’s welfare is ambiguous, relying on the parameters of country size, trade elasticity, and the level of variable trade costs.

The result is summarized as follows.

**Proposition 2.** In the presence of mobile capital, the change in real income associated with a change in variable trade costs can be calculated from Equation (26). Conditional on the share of expenditure on domestic goods, the larger country gains more from trade in the presence of mobile capital. For the smaller country, the decrease in wage-rent ratio and the increase in returns on capital affect its welfare gains from trade in opposite directions, resulting in an ambiguous net effect.

### 5.3 Numerical exercises

In this subsection, we examine the welfare implications of mobile capital via numerical exercises. We choose $\theta = 0.6$ as the larger country (country 1)’s population share. The elasticity of substitution is set as $\sigma = 4$, which is consistent with the estimates reported in Bernard et al. (2003). The fixed cost of domestic production is normalized as $f_{ii} = f_d = 1(i = 1, 2)$. The sunk cost of entry is set as $f_e = 1$, since it does not affect the ratio of productivity cutoffs ($\varphi_{ij}/\varphi_{ii}$) ($j \neq i$) according to (17) and (20). Besides, we assign the variable trade cost as $\tau_{ij} = \tau = 2$ ($j \neq i$), and set the fixed cost of exporting as $f_{ij} = f_x = 1(j \neq i)$.

We let the productivity distribution be symmetric between countries while choosing three different values of the shape parameter $\kappa$. First, we set $\kappa = 5$, which is also the
partial elasticity of trade flow with respect to variable trade cost $\tau$ specified in Costinot and Rodríguez-Clare (2014). Second, we set a higher value of $\kappa = 8$ so that the (absolute) value of the trade elasticity is larger. In this case, each entrant is more likely to get a productivity draw of small values. As a result, firm selection into exporting is tougher. Finally, following di Giovanni and Levchenko (2013), we set a smaller value of $\kappa = 3$, which shuts down the channel of firm selection into exporting and leads to a smaller (absolute) value of the trade elasticity.\footnote{Regarding the trade elasticity parameters, Simonovska and Waugh (2014)'s estimates range between 2.79 and 4.46, whereas Eaton and Kortum (2002)'s estimates range between 2.44 and 12.86. By setting different values of $\kappa$, we link the trade elasticities to the mechanism of firm selection into exporting, in the sense that $\kappa$ affects both the proportion of exporting firms and the responses of trade flows.}

### 5.3.1 Firm heterogeneity and the welfare effects of trade with mobile capital

Equation (26) has shown that the presence of mobile capital affects a country’s welfare via the relative factor price adjustment. We now examine whether this channel of adjustment is quantitatively important for the welfare effects of trade. To do this, we consider the welfare gains from trade calculated as $(V^T_j - V^A_j)/V^A_j$. This term measures the percentage change in real income from autarky to trade. Table 5.3.1 documents the gains from trade, together with welfare impacts of domestic trade share changes and relative factor price changes, under different cost functions. Panel A corresponds to the baseline cost function, while Panel B shows the welfare results in the composite factor setup.

We obtain two sets of main results from Table 5.3.1. First, the welfare impact of change in relative factor prices is quantitatively important. When $\kappa = 5$, country 1’s welfare increases by 0.53 percent, among which the reduction in domestic trade share raises welfare by 0.47 percent, and the increase in wage-rent ratio leads to an additional 0.06 percent increase in welfare.\footnote{Autarky leads to the absence of goods trade and capital flow between countries. With $L_j = K_j$, $w^A_j/r^A_j = \sigma - 1$ holds under the baseline cost function, and $w^A_j/r^A_j = \alpha/(1 - \alpha)$ holds under the Cobb-Douglas composite factor setup.} Relative factor price hence accounts for about 11.7 percent of country 1’s welfare adjustment. The smaller country (country 2) gets a larger value of gains from trade (0.73 percent). The decline in wage-rent ratio, however, decreases country 2’s welfare by 0.1 percent, which accounts for about 12.1 percent of the net increase in welfare. Overall, the change in a country’s relative factor price is related to the direction of capital flows, and the contribution of mobile capital to the welfare gains
Table 1: Gains from trade

<table>
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<th>( \kappa )</th>
<th>Country 1</th>
<th></th>
<th></th>
<th>Country 2</th>
<th></th>
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<td>Decomposition</td>
<td>% of contribution</td>
<td>Total effect</td>
<td>Decomposition</td>
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<td>Gains from trade</td>
<td>Trade share price</td>
<td>Trade share price</td>
<td>Gains from trade</td>
<td>Trade share price</td>
<td>Trade share Price</td>
<td></td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
</tr>
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<td>3.179%</td>
<td>2.983%</td>
<td>0.195%</td>
<td>93.85%</td>
<td>6.15%</td>
<td>4.863%</td>
<td>5.145%</td>
</tr>
<tr>
<td>5</td>
<td>0.527%</td>
<td>0.465%</td>
<td>0.062%</td>
<td>88.29%</td>
<td>11.71%</td>
<td>0.726%</td>
<td>0.814%</td>
</tr>
<tr>
<td>8</td>
<td>0.046%</td>
<td>0.035%</td>
<td>0.011%</td>
<td>75.82%</td>
<td>24.18%</td>
<td>0.053%</td>
<td>0.069%</td>
</tr>
</tbody>
</table>

Panel A: Baseline setup \( (TC_{ij} = r_i f_{ij} + w_i \tau_{ij} q_{ij} / \varphi) \)

Panel B: Composite factor setup \( (TC_{ij} = w_i r_i^{1-\alpha}(f_{ij} + \tau_{ij} q_{ij} / \varphi) \) with \( \alpha = 0.67 \)

Note: The table shows the welfare impacts of trade under mobile capital and firm heterogeneity. Column 1 lists the shape parameter \( \kappa \) of the productivity distribution. Columns 2 and 7 report the percentage changes in welfare from autarky to trade. Columns 3 and 8 report the adjustments of welfare to changes in domestic trade share, and columns 4 and 9 report the adjustments of welfare to changes in wage-rent ratio. Columns 5 and 6 report the contributions of domestic trade share and relative factor price to country 1’s welfare gains from trade. Columns 10 and 11 report the contributions of domestic trade share and relative factor price to country 2’s welfare gains from trade. In addition, Panel B presents the results for the setup where the composite factor is used in both entry and production.

Second, the shape parameter \( \kappa \) affects the values of welfare gains from trade and the relative contribution of factor price adjustment. In the baseline cost function, if \( \kappa = 3 \), the value of welfare gains from trade for country 1 increases to 3.2 percent, and the change in wage-rent ratio accounts for 6 percent among the welfare improvement. When \( \kappa = 8 \), the value of country 1’s welfare gains from trade decreases to 0.5 percent, among which the wage-rent ratio contributes 24 percent. Similarly, when \( \kappa \) increases from 5 to 8, the value of country 2’s welfare gains from trade drops to 0.05 percent, and the decline in local wage-rent ratio (negatively) contributes 31 percent to the welfare change.

Intuitively, the shape parameter \( \kappa \) affects the welfare impacts of trade and capital mobility via the channel of firm selection into export market. When \( \kappa \) is larger, entrants are more likely to get low productivity draws, which decreases the ratio of exporting firms and leads to a smaller share of exports (imports) among each country’s value of production.

measure is sizeable.
output (expenditure).\textsuperscript{28} The welfare-improving effect of trade is hence weakened when firms find it less likely to engage in exporting. With mobile capital, the relative factor price’s contribution to welfare is also dependent on the parameter $\kappa$ since it affects the imbalances of goods trade.\textsuperscript{29} Specifically, under our baseline cost function, the value of country 1’s export ($X_{12}$) is about 13 percent higher than that of country 2’s export ($X_{21}$) when $\kappa = 5$, and increasing $\kappa$ to 8 raises the ratio of $X_{12}/X_{21}$ to 1.32. In other words, mobile capital strengthens the large country’s market size advantage, and this effect is magnified when $\kappa$ is larger so that firms are more difficult to engage in exporting and countries are relatively less integrated through trade. Correspondingly, the change in relative factor price plays a larger role in each country’s welfare adjustment.

5.3.2 Welfare implications of capital mobility and factor endowments

We now explore how capital mobility affects the welfare gains from trade. We compare the welfare terms between two cases: one with perfect capital mobility and the other with capital immobility.\textsuperscript{30} The immobility of capital changes the equilibrium in two ways. First, it mutes the firm delocation effect so that entry by firms does not respond to changes in trade costs. The mass of entrants is always proportional to the endowment of capital in a country.\textsuperscript{31} Second, trade in goods is now balanced. Hence a country’s total revenue equals its total income earned by the factors of production.

In addition, a key assumption made thus far is that countries differ only in population sizes. While it helps us to focus on the role of increasing returns to scale, this assumption excludes the existence of differences in relative factor endowments between countries.

\textsuperscript{28}Under the baseline cost function, the proportion of exporting firms in country 1 is 100% if $\kappa = 3$, 2.66% if $\kappa = 5$, and 0.36% if $\kappa = 8$; for country 2, the proportion of exporting firms is 100% if $\kappa = 3$, 3.67% if $\kappa = 5$, and 0.42% if $\kappa = 8$. Similar results hold under the composite factor setup.

\textsuperscript{29}In the case of perfectly mobile capital, the worldwide rental rate of capital is a weighted sum of wage rates across countries. Hence the wage-rent ratios are affected by the asymmetries in wage rate between countries, which is related to the direction of goods trade and capital flows.

\textsuperscript{30}If capital mobility is not perfect, we can introduce an additional iceberg-type cost when capital flows from one country to the other. We do not perform this exercise directly. The welfare gains from trade with imperfect capital mobility lie between the gains with capital immobility and the gains with perfect capital mobility.

\textsuperscript{31}With capital immobility, the equilibrium mass of firms $M^e_i$ is $M^e_i = \frac{(\sigma - 1)K_i}{\sigma \alpha j e}$ in the baseline setup and $M^e_i = \frac{(\sigma - 1) L^\alpha_i K^{1-\alpha}_i}{\alpha \alpha j e (1-\alpha)^1-\alpha}$ in the Cobb-Douglas composite factor setup. Hence $M^e_i$ does not vary with variable trade cost, and $M^e_i/M^e_j = K_i/K_j$ and $M^e_i/M^e_j = (L^\alpha_i K^{1-\alpha}_i)/(L^\alpha_j K^{1-\alpha}_j)$ hold.
Between 1975 and 2010, for instance, the average capital–labour ratio in Germany was around 20% lower than that in the US. Similar patterns are observed in other countries such as Denmark or Spain. Additionally, although the model predicts that a large country is a net exporter of goods, some large countries, such as the US, actually run a deficit in goods trade. We are, thus, in a position to account for the possibility of a large country being a net importer in goods trade, via introducing country asymmetry in factor endowments.

Table 2 numerically evaluates the impact of the difference in relative factor endowments and capital mobility under the baseline cost function. The results for the setup of Cobb-Douglas composite factor are similar, which are reported in Table I.1 of Appendix I. Column 1 sets the shape parameter as $\kappa = 3, 5$ and $\kappa = 8$, respectively. We set the world endowment of capital and labour as $K = L = 1000$ and maintain country 1’s population share as $\theta \equiv L_1/L = 0.6$. Column 2, however, assigns different values of $(K_1/L_1)/(K_2/L_2)$ that reflect the asymmetry in relative factor endowment. We consider the long run equilibrium under scenarios of either capital mobility or capital immobility. Column 3 reports the imbalance in goods trade, as represented by the ratio of bilateral trade values ($X_{12}/X_{21}$). Columns 4 to 7 report the welfare implications of capital mobility in two aspects. First, in the case of mobile capital, columns 4 and 6 indicate the contribution of relative factor price change to each country’s welfare gains from trade. Second, columns 5 and 7 calculate the difference in the welfare gains from trade if we keep the endowments of each country fixed but change capital from being immobile to being mobile.

We obtain the following findings from Table 2. First, if the larger country is also abundant in capital, then it is possible that capital flows out and the larger country becomes a net importer of goods. This is shown by the negative correlation between $(K_1/L_1)/(K_2/L_2)$ (in column 2) and $X_{12}/X_{21}$ (in column 3). Note that trade in goods is balanced if capital is immobile, and the equilibrium rental rate in a country is inversely related to its capital–labour ratio. Hence, the rate of return to capital in a large country with abundant capital may be lower than that in a small country with scarce capital. With mobile capital, any difference in rental rates between countries is now counteracted.

---

32 Between 1990 and 2015, the average deficits of US trade in goods with Denmark, Finland, and Germany amounted to 2256, 1520, and 34556 millions of US dollars, respectively. See [https://www.census.gov/foreign-trade/balance/index.html](https://www.census.gov/foreign-trade/balance/index.html#F) for details of US trade in goods by country.

33 The other parameters of $f_d, f_e, f_x, \tau, \kappa$ and $\alpha$ are kept equal to the values assigned in Section 5.3.1.
Table 2: Welfare implications of capital mobility (baseline setup)

<table>
<thead>
<tr>
<th>κ</th>
<th>$K_1/L_1$</th>
<th>$K_2/L_2$</th>
<th>$X_{12}/X_{21}$</th>
<th>% contribution of factor price</th>
<th>% diff. in $V$</th>
<th>% contribution of factor price</th>
<th>% diff. in $V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.07</td>
<td>0.87</td>
<td>-13.23%</td>
<td>6.15%</td>
<td>3.26%</td>
<td>-5.81%</td>
<td>-2.57%</td>
</tr>
<tr>
<td>3</td>
<td>1.40</td>
<td>1.13</td>
<td>11.71%</td>
<td>31.19%</td>
<td>23.39%</td>
<td>-25.43%</td>
<td>-6.59%</td>
</tr>
<tr>
<td>5</td>
<td>0.61</td>
<td>0.15</td>
<td>-321.66%</td>
<td>65.30%</td>
<td>85.90%</td>
<td>-69.14%</td>
<td>-8.86%</td>
</tr>
<tr>
<td>8</td>
<td>1.78</td>
<td>1.32</td>
<td>24.18%</td>
<td>95.35%</td>
<td>442.24%</td>
<td>-225.48%</td>
<td>21.07%</td>
</tr>
</tbody>
</table>

Note: This table shows the welfare implications of capital mobility under the baseline cost function with different factor endowments. Column 1 lists the shape parameter $\kappa$ of the productivity distribution. Column 2 assigns different values of the factor endowment ratio between countries. Column 3 calculates the differences in export values for the case of mobile capital, while $X_{12}/X_{21} = 1$ holds if capital is immobile. Columns 4 and 6 report the contribution of relative factor price change to each country’s welfare gains from trade when capital is mobile. Columns 5 and 7 report the percentages of difference between the welfare gains from trade with capital mobility and those without capital mobility.

Second, factor endowment difference and firm selection into export market magnify the welfare impacts of change in factor prices. If the capital–labour ratio of country 1 is 1.25 times that of country 2, then the welfare adjustment to the change in wage-rent ratio accounts for 13 percent of each country’s welfare gains from trade when $\kappa = 3$, and the proportion increases to 52 percent for country 1 and 43 percent for country 2 when $\kappa = 5$. In each case, the role of relative factor price change is strengthened in the adjustment of a country’s welfare.

Finally, columns 5 and 7 reveal that the welfare consequence of a change in the capital mobility tends to be larger if the relative factor endowment ratios are more asymmetric. In the baseline model with symmetric factor endowments, introducing mobile capital

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34In this case, factor endowment asymmetry also generates a difference in the rental rates of capital under autarky, so that $r_j^A(j = 1, 2)$ varies between countries.
increases (decreases) welfare for the larger (smaller) country. For instance, if \( \kappa = 5 \), then the presence of mobile capital increases country 1’s welfare gains from trade by 7 percent and decreases country 2’s welfare gains from trade by 6 percent. When the capital–labour ratio of country 1 increases to 1.25 times that of country 2, the flow of capital from country 1 to country 2 now decreases country 1’s welfare gains from trade by 17 percent and increases country 2’s welfare gains from trade by 39 percent. Hence factor endowment difference magnifies the welfare implications of mobile capital. Interestingly, an outflow of capital may not necessarily lead to a loss of welfare: if the capital–labour ratio of country 2 (the smaller country) is three-quarters that of country 1, an outflow of capital still raises country 2’s gains from trade by about 21 percent when \( \kappa = 8 \).

6 Conclusion

This paper presents a highly tractable model that embeds mobile capital in a trade model with firm selection into export markets and country asymmetry in population sizes. It features interactions between trade in goods and capital reallocation between countries. Unlike the existing one-factor, one-sector trade models that generate the wage convergence results, we show that trade liberalization, as represented by decreasing variable trade costs, generates either a bell-shaped or a monotonically increasing wage gap. We obtain two results regarding the impact of selection into export markets on cross-country inequalities. First, trade liberalization results in an increasing wage gap if exporting involves a sufficiently higher fixed cost than does domestic production. Second, selection into export markets leads to a monotonically increasing asymmetry in the location of producing firms when trade is liberalized.

We complement the existing literature by studying the welfare effects of trade when mobile capital appears as a factor of production. We link the welfare implications of mobile capital to the directions of goods trade and capital flows between asymmetric countries. We identify two margins of welfare adjustment when trade is liberalized. Namely, welfare responds to changes in a country’s share of expenditure on domestic goods and in the relative factor price between labour and capital. Conditional on the share of expenditure on domestic goods, the welfare gains from trade are greater for the larger country in the presence of mobile capital. For the smaller country, the welfare implication of capital mobility is ambiguous. While the mobility of capital improves its allocation across coun-
tries, the resulting higher return to capital is particularly beneficial if a country is a net exporter in good trade and a net importer of capital services. Additionally, firm selection into exporting magnifies the relative contribution of factor price change among the welfare adjustment to trade liberalization.

One feature of our analysis is that the incomes from capital invested abroad are repatriated to the country in which the capital owners live. Hence, the country where the income from capital is spent may differ from the country where capital is used in production. This allows us to capture the imbalance of trade in goods and examine its welfare implications in a static setup. The welfare gains, however, may be different in a dynamic context where forces such as capital accumulation and technology diffusion matter. Another direction for future work is to consider a multi-country setup where countries differ in productivity distribution or access to world trade. This would allow us to apply cross-country data and further evaluate the quantitative welfare consequences of capital mobility and trade imbalance.

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Appendix A  Wage equation

According to (3), (6), (11) and (14), we rewrite the domestic productivity cutoff $\varphi_{ii}^{*}$ as

$$
(\varphi_{ii}^{*})^\kappa = \frac{\sigma f_{d}}{(\kappa - \sigma + 1)f_{e}X_{i}}(w_{i}L_{i} + w_{i}^{\kappa}w_{j}^{1-\kappa}L_{j}\Delta),
$$

(A.1)

where the total expenditure in country $i$ is equal to its total income: $X_{i} = (w_{i} + \bar{r})L_{i}$. Plugging (A.1) into (16), the wage equation is expressed as $F(w, \Delta) \equiv A_{2}(w)\Delta^{2} + A_{1}(w)\Delta + \ldots$.
\[ A_0(w) = 0, \text{ where } A_2(w) = \theta(\sigma - 1 + \theta)w - (1 - \theta)(\sigma - \theta), \ A_1(w) = \sigma[(1 - \theta)w^\kappa - \theta w^{1 - \kappa}], \text{ and } A_0(w) = \theta(1 - \theta)(w - 1). \]

**Appendix B  Proof of Lemma 1**

All functions \( A_2(w), A_1(w), \) and \( A_0(w) \) increase with \( w \), so \( F(w, \Delta) \) is an increasing function of \( w \). Since \( A_2(1) = (2\theta - 1)\sigma > 0, A_0(1) = 0, \) and \( A_1((\frac{\theta}{1 - \theta})^{\frac{1}{\kappa - 1}}) = 0, \) we have \( F((\frac{\theta}{1 - \theta})^{\frac{1}{\kappa - 1}}, \Delta) > 0 \) and \( F(1, \Delta) = (2\theta - 1)\sigma\Delta(\Delta - 1) \leq 0. \) Therefore, the wage equation has a unique solution \( w^* \) in \([1, (\frac{\theta}{1 - \theta})^{\frac{1}{\kappa - 1}}]\) for any given \( \Delta \in [0, 1] \). Another upper bound of \( w^* \), i.e., \( \Delta^{\frac{1}{\kappa}} \), comes from the positiveness of (17). Note that \( w^* > 1 \) holds when \( \Delta \in (0, 1). \)

To prove the monotone properties regarding \( \theta \), we calculate the partial derivatives of \( F \) with respect to \( \theta \). We have

\[
\frac{\partial F}{\partial \theta} = -(1 - \Delta^2)(2\theta - 1)(w - 1) - \Delta\sigma[w(w^{-\kappa} - \Delta) + w^{\kappa} - \Delta] \leq 0,
\]

where the inequalities hold strictly because \( w > 1 \) is true when \( \Delta \in (0, 1). \)

**Appendix C  Trade in goods**

The equilibrium wage rate \( w^* > 1 \) holds when \( 0 < \Delta < 1. \) Note that

\[
\left( w^{1 - 2\kappa} \frac{\theta}{1 - \theta} \frac{1 - \Delta w^\kappa}{1 - \Delta w^{-\kappa}} - 1 \right) \left( \frac{\theta w^{1 - \kappa} \Delta + (1 - \theta)\Delta w^{-\kappa}}{1 - \theta + \theta w^{1 - \kappa} \Delta} \right) = w^{1 - 2\kappa} \frac{\theta}{1 - \theta} \frac{1 - \Delta w^\kappa}{1 - \Delta w^{-\kappa}} - \frac{\theta w^{1 - 2\kappa} + (1 - \theta)w^{-\kappa} \Delta}{1 - \theta + \theta w^{1 - \kappa} \Delta}
\]

\[
= \left[ \frac{\theta w^{1 - 2\kappa} + (1 - \theta)w^{-\kappa} \Delta}{1 - \theta + \theta w^{1 - \kappa} \Delta} \right] \left( \frac{(w - 1)(1 - \theta + w\theta)}{w(\sigma - 1 + \theta) + 1 - \theta} \right) > 0,
\]

where the second equality is from (20). Thus,

\[
w^{1 - 2\kappa} \frac{\theta}{1 - \theta} \frac{1 - w^\kappa \Delta}{1 - \theta - w^{-\kappa} \Delta} > 1. \quad (C.1)
\]

The ratio of bilateral export value equals

\[
\frac{X_{12}}{X_{21}} = w^{1 - 2\kappa} \left( \frac{\theta}{1 - \theta} \right) (\varphi_{11}^* \varphi_{22}^*)^{\kappa} = w^{1 - 2\kappa} \left( \frac{\theta}{1 - \theta} \right) \frac{1 - w^\kappa \Delta}{1 - w^{-\kappa} \Delta} > 1,
\]

where the inequality results from (C.1). Thus, Country 1 is a net exporter of goods.
Appendix D  The curves in Figure 1

The capital mobility curve. When $w > 1$, the derivatives of (19) are calculated as follows:

$$
\begin{align*}
\frac{\partial \lambda}{\partial w} &= \frac{\theta(1 - \theta)w^{-\kappa}\{(\kappa - 1)\Delta(1 + w^{-\kappa}) + w^{-\kappa}[1 + (1 - 2\kappa)\Delta^2]\}}{[\Delta + \theta w^{1-2\kappa}\Delta - \theta\Delta - w^{-\kappa}(1 - \theta + \theta w)]^2} \\
&> \frac{\theta(1 - \theta)w^{-\kappa}\{2(\kappa - 1)\Delta w^{-\kappa} + w^{-\kappa}[1 + (1 - 2\kappa)\Delta^2]\}}{[\Delta + \theta w^{1-2\kappa}\Delta - \theta\Delta - w^{-\kappa}(1 - \theta + \theta w)]^2} \\
&= \frac{\theta(1 - \theta)w^{-\kappa}\{2(\kappa - 1)\Delta w^{-\kappa}(1 - \Delta) + w^{-\kappa}(1 - \Delta^2)\}}{[\Delta + \theta w^{1-2\kappa}\Delta - \theta\Delta - w^{-\kappa}(1 - \theta + \theta w)]^2} \\
&\geq 0,
\end{align*}
$$

Thus the capital mobility curve is positively sloped and shifts up with larger values of $\Delta$.

The balance of payment curve. We rewrite (22) as $\mathcal{J}(w, \lambda, \Delta) = 0$ where

$$
\mathcal{J}(w, \lambda, \Delta) \equiv \Delta\left[\theta w^{1-\kappa} - w^{\kappa-1}\frac{(1 - \theta)^2}{\theta} \frac{\lambda}{1 - \lambda}\right] - \theta(w - 1)\frac{1 - \theta + \theta w^{1-\kappa}\Delta}{\theta w + 1 - \theta}.
$$

It follows immediately that

$$
\begin{align*}
\frac{\partial \mathcal{J}(w, \lambda, \Delta)}{\partial \lambda} &= -\Delta w^{\kappa-1}w + \sigma - \theta)(1 - \theta)^2 \frac{(1 - \theta)^2}{\theta(1 - \lambda)^2} < 0, \\
\frac{\partial \mathcal{J}(w, \lambda, \Delta)}{\partial \Delta} &= \theta w^{1-\kappa} - w^{\kappa-1}(1 - \theta)^2 \frac{\lambda}{1 - \lambda} - \theta(w - 1)\frac{\theta w^{1-\kappa}}{\theta w + 1 - \theta} \\
&= \frac{\theta(w - 1)(1 - \theta)}{\Delta(\theta w + 1 - \theta)} > 0.
\end{align*}
$$

In addition,

$$
\begin{align*}
\frac{\partial \mathcal{J}(w, \lambda, \Delta)}{\partial w} &= (1 - \kappa)\Delta w^{-\kappa}\left[\theta\sigma + w^{2(\kappa-1)}(\theta(w - 1) + \sigma)(1 - \theta)^2\frac{\lambda}{\theta(1 - \lambda)}\right] \\
&\quad - \theta(1 - \theta + \theta w^{1-\kappa}\Delta) + \Delta\left[\theta w^{1-\kappa} - w^{\kappa-1}(1 - \theta)^2\frac{\lambda}{\theta(1 - \lambda)}\right].
\end{align*}
$$

The first term is negative if $\kappa > 1$ and $w > 1$. Because $\mathcal{F}(w, \lambda) = 0$, the last two terms can be rewritten as

$$
-\theta(1 - \theta + \theta w^{1-\kappa}\Delta) + \theta^2(w - 1)\frac{1 - \theta + \theta \Delta w^{1-\kappa}}{\theta w + \sigma - \theta}
$$

$$
= - (1 - \theta + \theta w^{1-\kappa}\Delta)\left(\frac{\theta\sigma}{\theta w + \sigma - \theta}\right) < 0.
$$

Therefore, it holds that

$$
\frac{d\lambda}{dw} = -\frac{\partial \mathcal{J}/\partial w}{\partial \mathcal{J}/\partial \lambda} < 0, \quad \text{and} \quad \frac{d\lambda}{d\Delta} = -\frac{\partial \mathcal{J}/\partial \Delta}{\partial \mathcal{J}/\partial \lambda} > 0.
$$
Appendix E  The relationship between \( \theta \) and productivity cutoffs

For the larger country, exporting always requires a higher productivity cutoff than does domestic production (\( \varphi_{11}^* < \varphi_{12}^* \)). However, for the smaller country, when market size asymmetry is sufficiently large, it is likely that the productivity cutoff for exporting is smaller than that for domestic production (\( \varphi_{22}^* > \varphi_{21}^* \)). From (16) and (17), we obtain

\[
\left( \frac{\varphi_{22}^*}{\varphi_{21}^*} \right)^\kappa = \frac{1 - w^{-\kappa} \Delta}{1 - w^{\kappa} \Delta} w^{\kappa \Lambda - \kappa}.
\]  

(E.1)

The RHS of (E.1) increases with \( w \) because

\[
\frac{d}{dw} \left( \frac{1 - w^{-\kappa} \Delta}{1 - w^{\kappa} \Delta} \right) = \frac{\kappa w^{\kappa-1}(1 - \Delta^2)}{(1 - w^{\kappa} \Delta)^2} \geq 0.
\]

By Lemma 1, the equilibrium wage rate \( w^* \) increases with \( \theta \). We use \( w(\theta) \) to denote the dependence. The RHS of (E.1) then increases with \( \theta \), so \( \varphi_{22}^* > \varphi_{21}^* \) holds when \( \theta \geq \hat{\theta} \), where the threshold value \( \hat{\theta} \) solves

\[
1 = \left[ \frac{1 - w(\hat{\theta})^{-\kappa} \Delta}{1 - w(\hat{\theta})^{\kappa} \Delta} \right] w(\hat{\theta})^{\kappa \Lambda - \kappa}.
\]  

(E.2)

Equations (E.2) and (20) determine the threshold level of \( (\hat{\theta}, w(\theta)) \) above which \( \varphi_{22}^* > \varphi_{21}^* \) holds.

Appendix F  Equilibrium in the composite factor setup

Under the two-country setup with composite inputs, we obtain the following set of equilibrium variables for any given value of \( w \):

\[
M^e = \frac{(\sigma - 1)}{\alpha \sigma \kappa} \frac{w^{1-\alpha} \theta + (1-\theta)}{\bar{r}^{1-\alpha} f_e} L, \quad \lambda^e = \frac{w^{1-\alpha} \theta}{w^{1-\alpha} \theta + (1-\theta)},
\]

\[
\varphi_{11} = \left\{ \frac{\sigma - 1}{\kappa - \sigma + 1} \frac{f_d}{f_e} \frac{\theta w + (1-\theta) \Delta w^{\frac{\alpha \sigma}{\sigma + 1}}}{\alpha (w+r)} \right\}^\frac{1}{\kappa},
\]

\[
\varphi_{22} = \left\{ \frac{\sigma - 1}{\kappa - \sigma + 1} \frac{f_d}{f_e} \frac{1 - \theta + \theta \Delta w^{\frac{1-\alpha \sigma}{\sigma}}}{\alpha (1-\theta)(1+r)} \right\}^\frac{1}{\kappa},
\]

\[
\varphi_{21} = \varphi_{11} \tau \left( \frac{f_x}{f_d} \right)^{\frac{1}{\sigma - 1}} w^{-\frac{\alpha \sigma}{\sigma - 1}}, \quad \varphi_{12} = \varphi_{22} \tau \left( \frac{f_x}{f_d} \right)^{\frac{1}{\sigma - 1}} w^{\frac{\alpha \sigma}{\sigma - 1}},
\]

\[
\bar{r} = \frac{(1-\alpha)[w \theta + (1-\theta)]}{\alpha}.
\]

33
Using the fact of
\[
\left( \frac{\varphi_{11}}{\varphi_{22}} \right)^{-\kappa} = \frac{1 - \Delta w^{-\frac{\alpha\kappa}{\sigma - 1}}}{1 - \Delta w^{-\frac{\alpha\kappa}{\sigma - 1}}},
\]
the equilibrium wage rate \( w^* \) is uniquely determined by the following equation:
\[
w^{\frac{\alpha\kappa}{\sigma - 1}} \{ 1 - 2\theta + \theta(w - 1)[\alpha(\theta - 1) - \theta] \} \Delta^2
+ [w\theta - w^{\frac{2\alpha\kappa}{\sigma - 1}} (1 - \theta)] \Delta + w^{\frac{\alpha\kappa}{\sigma - 1}} (w - 1)(1 - \alpha) \theta (\theta - 1) = 0.
\]

### Appendix G  Price index and trade share

We derive the price index \( P_j \) from the zero-profit cutoff condition (3):
\[
P_j = \frac{\sigma w_j}{(\sigma - 1)\varphi_{jj}^*} \left( \frac{X_j}{\sigma r_j f_d} \right)^{\frac{1}{\sigma}}. \tag{G.1}
\]

With mobile capital, \( r_j = \bar{r} \) holds where \( \bar{r} \) is given by (11). On the one hand, the total expenditure on domestic goods \( X_{jj} \) is equal to the total revenue from sales in the domestic market:
\[
X_{jj} = M^e_j (\varphi_{jj}^*)^{-\kappa} \frac{\sigma^\kappa}{(\kappa - \sigma + 1)} \bar{r} f_d.
\]

On the other hand, \( X_{jj} = \pi_{jj} X_j \) holds. Together with the first equality of (8) and the last equality of (14), we have
\[
\varphi_{jj}^* = \left[ \frac{\sigma - 1}{(\kappa - \sigma + 1)} \frac{R_j}{f_d \pi_{jj} X_j} \right]^\frac{1}{\kappa}. \tag{G.2}
\]

By use of (G.1) and (G.2), we get the expression of \( P_j \) in equation (24).

### Appendix H  Welfare in the composite factor setup

In the setup of Cobb-Douglas composite factor, the worldwide interest rate is
\[
\bar{r} = \left( \frac{1 - \alpha}{\alpha} \right) \sum_i w_i L_i = \left( \frac{1 - \alpha}{\alpha} \right) \frac{\sum_i w_i L_i}{L}
\]
in equilibrium. Hence the wage-rent ratio in country \( i \) is \( \tilde{w}_i = \frac{w_i L_i}{\sum_i w_i L_i / L} \). The price index expression \( P_j \) satisfies:
\[
P_j \propto \pi_{jj}^\frac{1}{\kappa} (w_j^\alpha \bar{r}^{1 - \alpha})^\frac{\sigma}{\sigma - 1} X_j^\frac{1}{\sigma} \left( \frac{1 + \tilde{w}_j}{\tilde{w}_j} \right)^\frac{1}{\kappa},
\]
where \( X_j = (w_j + \bar{r})L_j \) is the total expenditure in country \( j \) if \( L_j = K_j \). Therefore, the following expression holds for the per-capita real income \( V_j \):

\[
V_j = \frac{X_j}{L_j P_j} \propto \frac{L_j}{1 + \bar{w}_j} \left( \frac{1 + \bar{w}_j}{\bar{w}_j} \right)^{1-\frac{1}{\kappa}} \left( \frac{1 + \bar{w}_j}{\bar{w}_j} \right)^{\frac{\alpha}{1-\sigma}} \frac{L_j^{\frac{1}{\sigma-1}}}{\kappa}
\]

from which we get the welfare expression in Footnote 24.

**Appendix I Additional results**

Table I.1: Welfare implications of capital mobility (Composite factor setup)

<table>
<thead>
<tr>
<th>( \kappa )</th>
<th>( \frac{K_1/L_1}{K_2/L_2} )</th>
<th>( \frac{X_{12}}{X_{21}} )</th>
<th>% contribution of factor price</th>
<th>% diff. in ( V )</th>
<th>% contribution of factor price</th>
<th>% diff. in ( V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
</tr>
<tr>
<td>1</td>
<td>1.09</td>
<td>8.46%</td>
<td>4.56%</td>
<td>-8.22%</td>
<td>-3.59%</td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>0.85</td>
<td>-15.61%</td>
<td>-6.41%</td>
<td>15.00%</td>
<td>8.97%</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>1.52</td>
<td>36.58%</td>
<td>28.45%</td>
<td>-34.55%</td>
<td>-9.39%</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.17</td>
<td>14.904%</td>
<td>8.984%</td>
<td>-16.147%</td>
<td>-7.219%</td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>0.58</td>
<td>-62.34%</td>
<td>-20.11%</td>
<td>45.26%</td>
<td>41.00%</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>2.93</td>
<td>69.50%</td>
<td>97.04%</td>
<td>-95.73%</td>
<td>-15.56%</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.36</td>
<td>26.57%</td>
<td>17.66%</td>
<td>-35.01%</td>
<td>-14.28%</td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>0.15</td>
<td>-372.90%</td>
<td>-48.50%</td>
<td>85.93%</td>
<td>191.64%</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>19.10</td>
<td>95.42%</td>
<td>433.30%</td>
<td>-336.27%</td>
<td>-6.62%</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table shows the welfare implications of capital mobility under the composite factor setup. Column 1 lists the shape parameter \( \kappa \) of the productivity distribution. Column 2 assigns different values of factor endowment ratios between countries. Column 3 calculates the differences in export values for the case of mobile capital, while \( X_{12}/X_{21} = 1 \) holds if capital is immobile. Columns 4 and 6 report the contribution of relative factor price change to each country’s welfare gains from trade when capital is mobile. Columns 5 and 7 report the percentages of difference between the welfare gains from trade with capital mobility and those without capital mobility. The composite factor is used in both entry and production.
References


