Online Appendix (Not for Publication)

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1 Extended model

In this section, we extend our baseline model by considering an alternative setup where production and entry involves different usages of capital and labour. We first present the model extension and its equilibrium conditions. Then we show that our main results about the effects of trade liberalization on international inequalities and welfare gains are robust across extended model setups.

Specifically, suppose that entry uses the immobile labour input, while production jointly uses mobile capital and immobile labour in a Cobb-Douglas form. The sunk cost of entry in country i is $w_i f_e$, the fixed cost of exporting from i to j is $w_i^\alpha r_i^{1-\alpha} f_{ij}$, and the variable cost is $w_i^\alpha r_i^{1-\alpha}$ where the rental rate of capital, r_i, is common across countries if capital is perfectly mobile. The cost function is then written as

$$TC_{ij}(\varphi) = w_i^\alpha r_i^{1-\alpha} \left[f_{ij} + \frac{x_{ij}(\varphi)}{\varphi} \right].$$

1.1 Model equilibrium

We now consider the equilibrium conditions in the new setup for capital usages.

First, the condition of free entry and exit equates the sunk costs of entry to the ex ante expected profit. For country i, this free entry condition becomes

$$w_i f_e = \frac{\sigma - 1}{\kappa - \sigma + 1} \sum_j (\varphi_{ij}^*)^{-\kappa} w_i^\alpha r_i^{1-\alpha} f_{ij}. \tag{A-1}$$

Multiplying both sides by the mass of firms M_i^e, we obtain

$$M_i^e w_i f_e = \frac{\sigma - 1}{\kappa - \sigma + 1} \sum_j M_{ij} w_i^\alpha r_i^{1-\alpha} f_{ij} \tag{A-2}$$

where the LHS indicates the total payment for entry by all entrants in country i, the term $M_{ij} = M_i^e (\varphi_{ij}^*)^{-\kappa}$ indicates the mass of firms selling from country i to country j, the term $\sum_j M_{ij} w_i^\alpha r_i^{1-\alpha} f_{ij}$ collects the sum of payment for fixed inputs used by producing firms in country i (that may sell goods to any destination market j), and (A-2) implies a constant ratio between the total payment for entry and the sum of payment for fixed inputs in production due to the Pareto productivity distribution.

Second, at the economy wide, we see that the following equation holds for any country i

$$M_i^e w_i f_e + \sum_j M_{ij} w_i^\alpha r_i^{1-\alpha} f_{ij} + (\frac{\sigma - 1}{\sigma}) Y_i = Y_i \tag{A-3}$$

where the term Y_i on the HRS indicates the total value of firm output in country i, and the term $(\frac{\sigma-1}{\sigma})Y_i$ indicates the total payment for variable inputs in production. Given the CES utility and constant markup pricing, the variable costs of production is a constant fraction $(\frac{\sigma-1}{\sigma})$ of firm revenue. Equation (A-3) says that the total revenue Y_i compensates the total sum of payment of entry costs (the first term on the LHS), the fixed costs in production and exporting (the second term on the LHS), and the variable costs (the third term on the LHS).

Combining (A-2) and (A-3), the equilibrium mass of entrants M_i^e is solved as

$$M_i^e = \frac{\sigma - 1}{\sigma\kappa} \frac{Y_i}{w_i f_e}, \tag{A-4}$$

which increases with the total value of firm revenue Y_i and decreases with the sunk cost of entry $w_i f_e$.

Next, we consider the equilibrium conditions for each factor market. For the labour market, given the Cobb-Douglas form of the inputs used in production, we have

$$w_i L_i = M_i^e w_i f_e + \alpha(\frac{\kappa - \sigma + 1}{\sigma - 1}) M_i^e w_i f_e + \alpha(\frac{\sigma - 1}{\sigma}) Y_i, \tag{A-5}$$

where the three terms on the RHS indicates wage payment for entry, fixed costs of production, and variable costs in production, respectively. Together with (A-4), we can show that the labour payment in country i is a constant fraction of the total firm revenue:

$$w_i L_i = \left[\alpha + \left(\frac{\sigma-1}{\sigma\kappa}\right)(1-\alpha)\right] Y_i. \tag{A-6}$$

For the capital market, since the total firm revenue is completely remunerated for capital and labour payments in entry and production, the capital payment in country i is also proportionate to the total firm revenue. Moreover, the mobility of capital equalizes the rental rate of capital across countries, so that $r_i = \bar{r}$ holds and the equilibrium rental rate $\bar{r}$ satisfies the condition that

$$\bar{r}\sum_i K_i = (1-\alpha)\left(\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}\right)\sum_i Y_i, \tag{A-7}$$

where $\bar{r}\sum_i K_i$ is the aggregate payment to capital inputs across countries, and $\sum_i Y_i$ is the aggregate firm revenue across countries. Equation (A-7) uses the condition of market clearing at the world level where the total payment to capital is equal to $\bar{r}\sum_i K_i$ when capital is mobile.

Equations (A-6) and (A-7) then indicate that the equilibrium rental rate $\bar{r}$ is proportionate to the weighted sum of wage rates across countries where the weights are population sizes of each country.

$$r_i = \bar{r} = \frac{(1-\alpha)(\sigma\kappa-\sigma+1)}{\sigma\kappa-(1-\alpha)(\sigma\kappa-\sigma+1)}\frac{\sum_i w_i L_i}{\sum_i K_i}. \tag{A-8}$$

Note that equation (A-8) bears resemblance to equation (11) in our baseline setup. [1]

Now we consider the expression for trade shares across countries. Similar to the analysis in our baseline model, we can derive expressions for trade shares

$$\pi_{ij} = \frac{X_{ij}}{X_j} = \frac{L_i \tau_{ij}^{-\kappa}\left(w_i^{\alpha}\right)^{\frac{\sigma-1-\sigma\kappa}{\sigma-1}} f_{ij}^{\frac{\sigma-1-\kappa}{\sigma-1}}}{\sum_s L_s \tau_{sj}^{-\kappa}\left(w_s^{\alpha}\right)^{\frac{\sigma-1-\sigma\kappa}{\sigma-1}} f_{sj}^{\frac{\sigma-1-\kappa}{\sigma-1}}},$$

[1] Meanwhile the wage-rent ratio $w_i/\bar{r}$ increases with country i's wage rate w_i. Our welfare analysis quantifies the importance of adjustment in relative factor prices for the welfare effects of trade. By contrast, if capital is immobile across countries, then rental rates are not equalized, and for country i, the condition $r_i K_i = (1-\alpha)(\frac{\sigma\kappa-\sigma+1}{\sigma\kappa})Y_i$ holds. In this case, the wage-rent ratio w_i/r_i is constant, and there is no role of relative factor prices in the welfare impacts of trade. This result is identical to other model setups in our paper.

and combine them with the expenditure expression $X_j = w_j L_j + \bar{r} K_j$ to derive total firm revenue in country i as

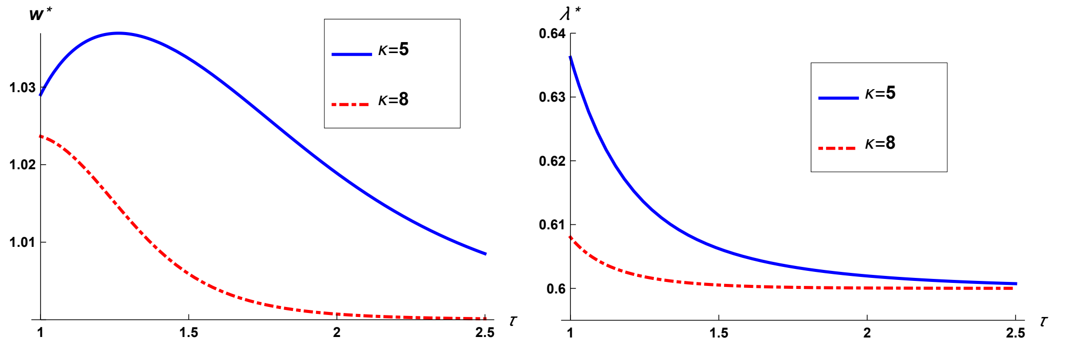
$$Y_i = \sum_j \frac{L_i \tau_{ij}^{-\kappa} \left(w_i^{\alpha}\right)^{\frac{\sigma-1-\sigma\kappa}{\sigma-1}} f_{ij}^{\frac{\sigma-1-\kappa}{\sigma-1}}}{\sum_s L_s \tau_{sj}^{-\kappa} \left(w_s^{\alpha}\right)^{\frac{\sigma-1-\sigma\kappa}{\sigma-1}} f_{sj}^{\frac{\sigma-1-\kappa}{\sigma-1}}} \left(w_j L_j + \bar{r} K_j\right), \tag{A-9}$$

where $\bar{r}$ is from (A-8).

Together with (A-6) from which the total firm revenue Y_i can be rewritten as a function of $w_i L_i$, we obtain an equation system that solves the equilibrium wage rates for each country (w_i^*) for any given values of factor endowments $\{L_i, K_i\}$, trade costs $\{\tau, f_x/f_d\}$ and other parameters like f_e, κ and σ. Once we get the solutions of the equilibrium wage rates (w_i^*), all the other endogenous variables in equilibrium like the cutoff productivities $\{\varphi_{ij}^*\}$, the mass of entrants $\{M_i^{e*}\}$, and the mass of producing firms that supply goods from country i to j $\{M_{ij}^*\}$ can be solved as functions of the equilibrium wages.

1.2 Trade liberalization and international inequalities

Using equations (A-8) and (A-9), we now consider a setup with two countries that differ in population sizes while being symmetric in other aspects, as what we have done in the baseline model. Using the same notations, now we examine whether our results on the effects if trade liberalization on international inequalities change under different usages of capital. Figure A-1 reveals that the between-country wage gap is bell shaped when the shape parameter is κ=5 while being monotonically increasing when the shape parameter is $\kappa = 8$. By contrast, for the differences in firm shares, the larger country's share is monotonically increasing in both cases. Hence the spatial inequalities in wages and firm allocations are differently affected by trade liberalization. Combining with the results in our baseline model setup and in the extended setup with Cobb-Douglas composite of capital and labour, the results of Proposition 1 are robust across different setups of capital usages.



(a) The larger country's relative wage (b) The larger country's firm share

Figure A-1: The effects of trade liberalization in the extended model setup

Note: In the extended model setup, both capital and labour are used in the production function, while entry requires only labour input only. The sunk cost of entry is $w_i f_e$, and the cost function for production is $TC_{ij}(\varphi) = w_i^{\alpha} r_i^{1-\alpha}(f_{ij} + \tau_{ij}\frac{x_{ij}(\varphi)}{\varphi})$.

1.3 Welfare analysis

We have shown in the main text that the welfare effects of trade liberalization is affected by the presence of mobile capital through the channel of change in relative factor price (or, wage-rent ratio) in a country. Now we evaluate whether the importance of relative factor price for welfare gains from trade is affected by different usages of capital or not. In doing so, we evaluate the welfare gains from trade in the new setup of factor usages. The welfare expression is now rewritten as

$$V_i = \frac{X_j}{L_j} \propto \pi_{jj}^{-\frac{1}{\kappa}} \left[\widetilde{w}_j^{1-\alpha} L_j + \widetilde{w}_j^{-\alpha} K_j\right]^{\frac{\sigma\kappa-\sigma+1}{\kappa(\sigma-1)}} L_j^{\frac{1}{\kappa}-1}, \tag{A-10}$$

which indicates that when capital is mobile, the welfare of a country responds to two margins of adjustment: one is the change in domestic expenditure share π_{jj} (as shown in the ACR welfare formula), the other is the change in relative factor prices $\widetilde{w}_j$.[2]

Now we numerically examine how the welfare impact of change in relative factor price is affected by the usages of different factor usages. With the welfare expression in (A-10), Table A-1 reports the welfare gains from trade in the new model setup. Columns (6) and (11) present the quantitative importance of change in relative factor prices for welfare. Similar to the results in the baseline model, we find that the adjustments of relative factor price are important to the welfare effects of trade, and the share of contribution does not change

[2]If $L_i = K_i$ so that countries are symmetric in relative factor endowments, then the welfare expression is simplified as $V_j \propto \pi_{jj}^{-\frac{1}{\kappa}} (\widetilde{w}_j^{1-\alpha} + \widetilde{w}_j^{-\alpha})^{\frac{\sigma\kappa-\sigma+1}{\kappa(\sigma-1)}} L_j^{\frac{\sigma}{\sigma-1}}$.

Table A-1: Gains from trade in the extended model setup

	Country 1					Country 2				
	Total effect	Decomposition		% of contribution		Total effect	Decomposition		% of contribution	
κ	Gains from trade	Trade share	Factor price	Trade share	Factor price	Gains from trade	Trade share	Factor price	Trade share	Factor Price
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Cost function: $TC_{ij} = w_i^{\alpha} r_i^{1-\alpha}(f_{ij} + \tau_{ij} q_{ij}/\varphi)$, entry cost: $w_i f_e$										
3	3.177%	2.984%	0.193%	93.92%	6.08%	4.864%	5.143%	-0.279%	105.74%	-5.74%
5	0.531%	0.462%	0.069%	87.05%	12.95%	0.721%	0.819%	-0.098%	113.65%	-13.63%
8	0.046%	0.034%	0.012%	74.25%	25.75%	0.052%	0.069%	-0.017%	133.44%	-33.44%

Note: The table shows the welfare impacts of trade under mobile capital and firm heterogeneity. Column 1 lists the shape parameter κ of the productivity distribution. Columns 2 and 7 report the percentage changes in welfare from autarky to trade. Columns 3 and 8 report the adjustments of welfare to changes in domestic trade share, and columns 4 and 9 report the adjustments of welfare to changes in wage-rent ratio. Columns 5 and 6 report the contributions of domestic trade share and relative factor price to country 1's welfare gains from trade. Columns 10 and 11 report the contributions of domestic trade share and relative factor price to country 2's welfare gains from trade.

much across different setups of capital usage. For instance, when the parameter of firm heterogeneity is $\kappa = 5$, then for the larger country (country 1), the welfare adjustment to change in relative factor price accounts for about 12.95% of the country's gains from trade. For the smaller country (country 2), as capital flows out, the change in relative factor price now decreases the country's gains from trade by about 13.63%. Note that all these values are close to those in Table 1 of the main text, and the results of both tables are derived from identical settings of exogenous parameter set of $\{\theta, \kappa, \sigma, f_x/f_d, f_e\}$, which implies that our welfare analysis of the role of relative factor prices is not affected by different usages of production factors.

Similarly, we can show that the welfare consequences of capital mobility (versus capital immobility) do not change across different setups of capital usage. Specifically, we replicate the exercise in Table 2 of the main text for the extended model setup, and report the results in Table A-2. Comparing the results in both tables, we find that the welfare consequences of relative factor price (reported in columns (4) and (7)) and capital mobility (reported in columns (5) and (7)) are not quantitatively affected by the usage of different factors.

Table A-2: Welfare implications of capital mobility (extended model setup)

			Country 1		Country 2	
κ	$\frac{K_1/L_1}{K_2/L_2}$	$\frac{X_{12}}{X_{21}}$	% contribution of factor price	% diff. in V	% contribution of factor price	% diff. in V
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	1	1.07	6.08%	4.29%	-5.74%	-3.51%
3	1.25	0.87	-13.14%	-7.52%	13.47%	10.58%
	0.75	1.40	30.99%	29.96%	-25.15%	-11.24%
	1	1.15	12.95%	7.69%	-13.65%	-6.18%
5	1.25	0.59	-56.66%	-20.54%	44.18%	44.03%
	0.75	2.69	67.24%	101.05%	-79.15%	-15.74%
	1	1.34	25.75%	17.53%	-33.44%	-14.13%
8	1.25	0.15	-358.55%	-47.92%	86.47%	209.36%
	0.75	18.94	95.47%	468.97%	-290.43%	-1.11%

Note: This table shows the welfare implications of capital mobility under the baseline cost function with different factor endowments. Column 1 lists the shape parameter κ of the productivity distribution. Column 2 assigns different values of the factor endowment ratio between countries. Column 3 calculates the differences in export values for the case of mobile capital, while $X_{12}/X_{21} = 1$ holds if capital is immobile. Columns 4 and 6 report the contribution of relative factor price change to each country's welfare gains from trade when capital is mobile. Columns 5 and 7 report the percentages of difference between the welfare gains from trade with capital mobility and those without capital mobility.

2 Derivation of equation (22)

In the presence of mobile capital and immobile capital owners (who are workers in each country), for each country, the net export of goods should be equal to the amount of net capital inflows according to the balance of payment condition. Hence our two-country setup indicates that

$$X_{12} - X_{21} = M^e \lambda^e \bar{r}[f_e + (\varphi_{11}^*)^{-\kappa} f_d + (\varphi_{12}^*)^{-\kappa} f_x] - \bar{r} K_1 \tag{A-11}$$

should hold, which is equivalent to (13). The LHS of (A-11) expresses country 1's net export of goods as

$$\begin{aligned} X_{12} - X_{21} &= M^e \frac{\sigma\kappa}{\kappa - \sigma + 1} \bar{r} f_x [\lambda^e (\varphi_{12}^*)^{-\kappa} - (1 - \lambda^e)(\varphi_{21}^*)^{-\kappa}] \\ &= M^e \frac{\sigma\kappa}{\kappa - \sigma + 1} \bar{r} f_x \Big[\lambda^e (\Lambda w)^{-\kappa} (\varphi_{22}^*)^{-\kappa} - (1 - \lambda^e)(\frac{\Lambda}{w})^{-\kappa} (\varphi_{11}^*)^{-\kappa}\Big], \end{aligned} \tag{A-12}$$

where we use equation (16) that relates cutoff productivities of exporting firms to the domestic cutoff productivities in the other country. Meanwhile, the RHS of (A-11) indicates the net inflow of capital into country 1, measured by the difference between the demand for capital from local firms and the endowment of capital owned by local workers:

$$M^e\lambda^e\bar{r}[f_e+(\varphi_{11}^*)^{-\kappa}f_d+(\varphi_{12}^*)^{-\kappa}f_x]-\bar{r}K_1=M^e\lambda^e\frac{\kappa}{\sigma-1}\bar{r}f_e-\bar{r}\theta K, \tag{A-13}$$

according to country 1's free entry condition under the Pareto distribution.

Substituting the expressions of M^e in equation (12) and λ^e in (15) into (A-12) and (A-13), equation (A-11) is rewritten as

$$\frac{\sigma(\sigma-1)}{\kappa-\sigma+1}\frac{f_x}{f_e}(\varphi_{22}^*)^{-\kappa}\Big[\theta w(\Lambda w)^{-\kappa}-(1-\theta)\Big(\frac{\Lambda}{w}\Big)^{-\kappa}\Big(\frac{\varphi_{11}^*}{\varphi_{22}^*}\Big)^{-\kappa}\Big]=\theta(1-\theta)(w-1). \tag{A-14}$$

Now we write φ_{22}^* and $\varphi_{11}^*/\varphi_{22}^*$ in (A-14) as functions of w and λ. First, for the cutoff productivity φ_{22}^*, combining the cutoff productivity condition in (3), the price index expression in (6), the worldwide mass of firms in (12) and the relationship of cutoff productivities in (16), we obtain

$$(\varphi_{22}^*)^{\kappa}=\frac{f_d}{f_e}\frac{\sigma(\sigma-1)}{(\kappa-\sigma+1)}\frac{\bar{r}}{(1-\theta)(1+\bar{r})}\left[(1-\lambda^e)+\lambda^e\Delta w^{-\kappa}\right].$$

Substituting the equilibrium rental rate $\bar{r}$ in (11) and equilibrium entrant share λ^e in (15) into the above equation, the expression of $(\varphi_{22}^*)^{-\kappa}$ is then rewritten as a function of w:

$$(\varphi_{22}^*)^{-\kappa}=\frac{\kappa-\sigma+1}{\sigma(\sigma-1)}\frac{f_e}{f_d}\frac{(1-\theta)(\theta w+\sigma-\theta)}{(1-\theta)+\theta w^{1-\kappa}\Delta}. \tag{A-15}$$

Meanwhile, the ratio of cutoff productivities $(\varphi_{11}^*/\varphi_{22}^*)$ is a function of λ and w according to equations (15) and (18):

$$\Big(\frac{\varphi_{11}^*}{\varphi_{22}^*}\Big)^{-\kappa}=\frac{\lambda}{1-\lambda}\frac{1-\lambda^e}{\lambda^e}=\frac{\lambda}{1-\lambda}\frac{1-\theta}{w\theta}. \tag{A-16}$$

Substituting (A-15) and (A-16) into (A-14), we then get the expression of the balance of payment condition in equation (22).

3 Empirical facts

Our theoretical analysis has the result that when some production factor like capital is internationally mobile, it is likely that the larger country's size advantage is amplified, which implies an increasing wage gap between countries. Meanwhile, depending on the shape parameters of firm heterogeneity, the international wage difference is either bell shaped or monotonically increasing when trade in goods is liberalized.

Figure A-2 provides the empirical evidence. For instance, despite gradual reductions in the bilateral trade cost, the per-capita income difference between Canada and the US has continued to increase since the 1980s. Meanwhile, in countries like Germany or Finland, it is far from obvious that trade costs with the US is negatively related to the income differences in a monotonic way.

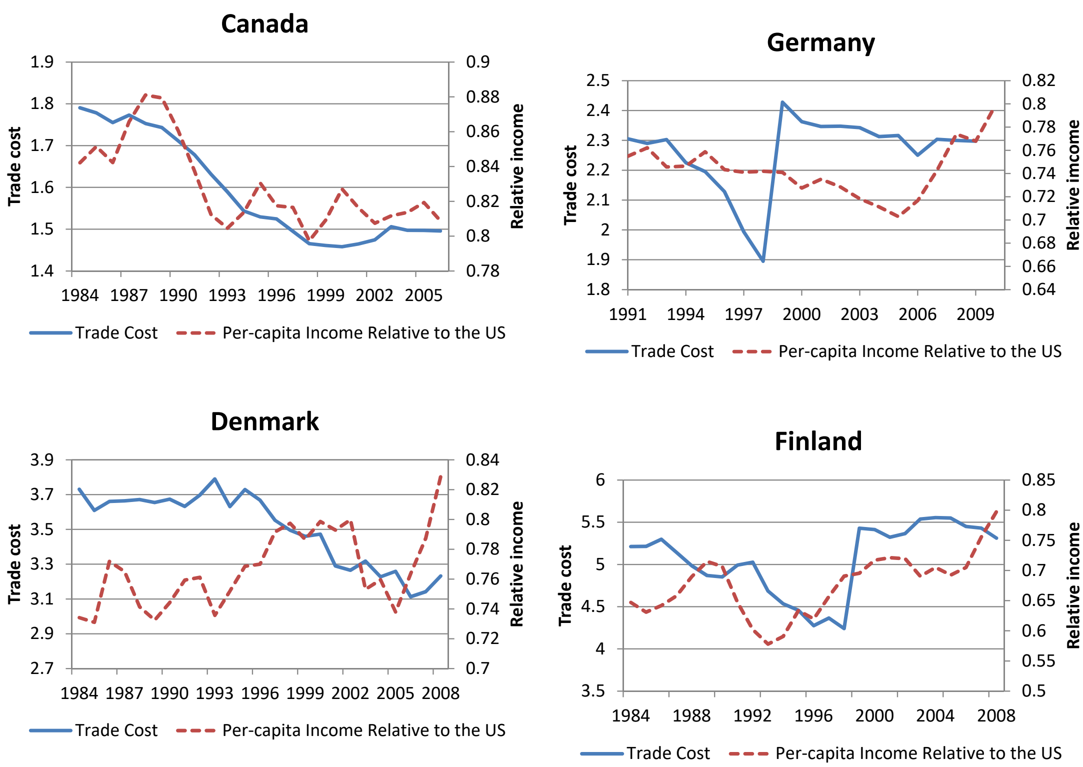


Figure A-2: Trade cost and the per capita income difference

Notes: Trade cost estimation follows Head and Ries (2001) and Novy (2013). Namely, given values of manufactured goods that country j imports from country i, X_{ij}, domestic trade values, X_{ii}, and elasticity of trade values with respect to trade cost, ε, the bilateral trade cost measure is $\tau_{ij} = \left(\frac{X_{ij}X_{ji}}{X_{ii}X_{jj}}\right)^{1/(2\varepsilon)}$. Domestic trade values are calculated as $X_{ii} = Y_i - EX_i$ where Y_i is the value of gross manufacturing production and EX_i is the value of exports. Following Simonovska and Waugh (2014), the trade cost elasticity is set as $\varepsilon = -4$.

Sources: Gross manufacturing production data is from OECD STAN Database for Structural Analysis; Export and import values are from NBER-UN trade data. Per capita income data is from Penn World Tables 8.1.

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